Day 159 **IGCSE 10-a-day: Model Solutions**

1 Area of a triangle using sine

In triangle PQR, PQ = 12 cm and angle PQR = 135°.
The area of the triangle is $54\sqrt{2}$ cm^2.
Work out the length of QR.
Do not use a calculator.

Area = $\frac{1}{2} \times PQ \times QR \times \sin Q$

$54\sqrt{2} = \frac{1}{2} \times 12 \times QR \times \sin 135°$

$\sin 135° = \frac{\sqrt{2}}{2}$, so $54\sqrt{2} = 3\sqrt{2} \times QR$

$QR = 54\sqrt{2} \div 3\sqrt{2} = 18$ cm

Answer: 18 cm

2 Point with a given gradient (quadratic)

A curve has equation $y = 3x^2 + 5x + 9$.
The gradient at the point P on the curve is -19.
Find the coordinates of P.

$\frac{dy}{dx} = 6x + 5$

$6x + 5 = -19$, so $6x = -24$

$x = -4$

$y = 37$

Answer: $(-4, 37)$

3 Find unknown constants in a function

f(x) = ax + b, where a and b are constants.
$f(-3) = 5$ and $f^{-1}(-16) = 4$.
Find the value of a and the value of b.

$f^{-1}(-16) = 4$ means $f(4) = -16$

$-3a + b = 5$ and $4a + b = -16$

Subtract: $-7a = 21$, so $a = -3$

$b = 5 - (-3) \times (-3) = -4$

Answer: $a = -3$, $b = -4$

4 Laws of indices with algebra

Simplify $(8x^6y^9)^{\frac{2}{3}}$

Raise each part to the power $\frac{2}{3}$

$8^{\frac{2}{3}} = (\sqrt[3]{8})^2 = 2^2 = 4$

$(x^6)^{\frac{2}{3}} = x^4$ and $(y^9)^{\frac{2}{3}} = y^6$

$= 4x^4y^6$

Answer: $4x^4y^6$

5 Compound interest: how many years

£2,500 is invested at 10% per year compound interest.
Work out the least whole number of years after which the investment is worth more than £5,000. (Calculator)

Value after n years = 2500×1.1^n

After 7 years: £4,871.79 (less than £5,000)

After 8 years: £5,358.97 (more than £5,000)

Answer: 8 years

6 Direct proportion formulae

y is directly proportional to the cube of x.
When x = 1, y = 12.
(a) Find a formula for y in terms of x.
(b) Work out the positive value of x when y = 324.

(a) $y = kx^3$
$12 = k \times 1$, so $k = 12 \div 1 = 12$
$y = 12x^3$
(b) $324 = 12x^3$, so $x^3 = 324 \div 12 = 27$, giving $x = 3$
Answer: (a) $y = 12x^3$ (b) $x = 3$

7 Recasting solids

A solid cylinder of radius 8 cm and height 10 cm is melted down.
The metal is used to make identical solid spheres of radius 2 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder $= \pi \times 8^2 \times 10 = 640\pi$
Volume of one sphere $= \frac{4}{3} \times \pi \times 2^3 = \frac{32}{3}\pi$
Number of spheres $= 640 \div \frac{32}{3} = 60$
Answer: 60

8 Stationary points of a cubic

A curve has equation $y = -2x^3 - 15x^2 - 24x + 2$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = -6x^2 - 30x - 24$
Stationary points: $\frac{dy}{dx} = 0$, so $-6(x + 4)(x + 1) = 0$
$x = -4$ or $x = -1$
$y = -14$ when $x = -4$ and $y = 13$ when $x = -1$
Answer: (−4, −14) and (−1, 13)

9 Bounds of roots

The area of a square is 353 cm^2, correct to the nearest cm^2.
Work out the upper bound of the side length of the square.
Give your answer correct to 3 significant figures. (Calculator)

$352.5 \leq$ area < 353.5
Upper bound of side $= \sqrt{353.5}$
$= 18.8016... = 18.8$ (3 s.f.)
Answer: 18.8 cm

10 Area of triangle from a tangent

The tangent to the curve $y = -x^2 + 3x - 2$ at the point where x = 2 meets the axes at A and B.
Work out the area of triangle OAB, where O is the origin.

$y = 0$ and $\frac{dy}{dx} = -2x + 3$, so the gradient is -1
Tangent: $y = -x + 2$
y-axis: (0, 2). x-axis: $0 = -x + 2$ so $x = 2$
Area $= \frac{1}{2} \times 2 \times 2$
$= 2$
Answer: 2 square units