Day 158 **IGCSE 10-a-day: Model Solutions**

1 Gradient with fractional or negative powers

The equation of a curve is $y = -2x^2 + \frac{10}{x}$.

Work out the gradient of the curve at the point where x = 2.

$\frac{dy}{dx} = -4x - 10x^{-2}$

When x = 2: gradient = $-\frac{21}{2}$

Answer: $-\frac{21}{2}$

2 Bounds and accuracy

a = 8.96 and b = 3.38, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$8.955 \leq a < 8.965$; $3.375 \leq b < 3.385$
Lower bound = 8.955 × 3.375 = 30.223125
Upper bound = 8.965 × 3.385 = 30.346525
Both bounds round to 30 to 2 significant figures, but not to 3 significant figures
Answer: 30 (2 s.f.)

3 Solving quadratics by factorising

Solve $x^2 - x - 30 = 0$

Factorise: $(x + 5)(x - 6) = 0$
$x + 5 = 0$ or $x - 6 = 0$
$x = -5$ or $x = 6$
Answer: $x = -5$ or $x = 6$

4 Points satisfying inequalities

R is the region that satisfies all of these inequalities:
$y \leq -2x + 6$ $x + y < 10$ $x \geq 1$
Which of these points lie in R?
A(0, 1) B(1, 3) C(3, 7) D(5, 4) E(8, 9)

A(0, 1): $y \leq 6$: yes; $x + y = 1$: yes; $x = 0$: no
B(1, 3): $y \leq 4$: yes; $x + y = 4$: yes; $x = 1$: yes
C(3, 7): $y \leq 0$: no; $x + y = 10$: no; $x = 3$: yes
D(5, 4): $y \leq -4$: no; $x + y = 9$: yes; $x = 5$: yes
E(8, 9): $y \leq -10$: no; $x + y = 17$: no; $x = 8$: yes
In R: B
Answer: B

5 Area of a triangle using sine

Triangle ABC has AB = 17 cm and BC = 20 cm.
The area of triangle ABC is 85 cm^2.
Angle ABC is acute.
Work out the size of angle ABC.
Do not use a calculator.

Area = $\frac{1}{2} \times AB \times BC \times \sin B$

$85 = \frac{1}{2} \times 17 \times 20 \times \sin B$

$\sin B = 85 \div 170 = \frac{1}{2}$

$\sin 30° = \frac{1}{2}$, and B is acute, so B = 30°

Answer: 30°

6 Points with a given gradient on y = ax + b/x

A curve has equation $y = -3x - \frac{6}{x}$.

Find the coordinates of the points on the curve where the gradient is 3.

$\frac{dy}{dx} = -3 + \frac{6}{x^2}$

Set $\frac{dy}{dx} = 3$: $-6 = \frac{-6}{x^2}$

$x^2 = 1$, so $x = 1$ or $x = -1$

$y = -9$ when $x = 1$ and $y = 9$ when $x = -1$

Answer: (1, −9) and (−1, 9)

7 Bounds: finding an unknown length

A rectangle has area 27 cm^2, correct to 2 significant figures.

Its length is 6.2000000000000001776356839400250464677810668945312 5 cm, correct to 2 significant figures.

Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: $26.5 \leq A < 27.5$. Length: $6.150000000000000177635683940025046467781066894531 25 \leq l < 6.250000000000000177635683940025046467781066894531 25$

UB width = UB area ÷ LB length = $27.5 \div 6.150000000000000177635683940025046467781066894531 25 = 4.47$

Answer: 4.47 cm

8 Exact lengths and areas

A rectangle has area $(8 + 4\sqrt{3})$ cm^2 and length $(2 + 2\sqrt{3})$ cm.

Find the width of the rectangle.

Give your answer in the form $a + b\sqrt{3}$

Width = area ÷ length = $\frac{8 + 4\sqrt{3}}{2 + 2\sqrt{3}}$

Multiply top and bottom by $(2 - 2\sqrt{3})$:

Numerator: $(8 + 4\sqrt{3})(2 - 2\sqrt{3}) = -8 - 8\sqrt{3}$

Denominator: $4 - 12 = -8$

Width = $1 + \sqrt{3}$

Answer: 1 + $\sqrt{3}$ cm

9 Compound interest

Priya invests £11,000 for 6 years at 4.5% per year compound interest.

Work out the value of the investment at the end of the 6 years.

Give your answer to the nearest penny. (Calculator)

Multiplier = 1 + 4.5 ÷ 100 = 1.045

$11000 \times 1.045^6 = 14324.86$

To the nearest penny: £14,324.86

Answer: £14,324.86

10 Points with a given gradient (cubic)

The curve $y = 2x^3 - 3x^2 - 25x - 3$ has gradient 11 at two points.

Find the coordinates of these points.

$\frac{dy}{dx} = 6x^2 - 6x - 25$

$6x^2 - 6x - 25 = 11$, so $6x^2 - 6x - 36 = 0$

Divide by 6: $x^2 - x - 6 = 0$

$(x + 2)(x - 3) = 0$, so $x = -2$ or $x = 3$

Substitute into y: (−2, 19) and (3, −51)

Answer: (−2, 19) and (3, −51)