Day 157 **IGCSE 10-a-day: Model Solutions**

1 Income tax

In one country, income tax is worked out like this:
The first £12,000 of income is not taxed.
The next £30,000 is taxed at 20%.
All income above that is taxed at 40%.
Omar earns £47,000 in a year.
Work out how much income tax Omar pays.

Taxed at 20%: £30,000
20% of £30,000 = £6,000
Taxed at 40%: £47,000 − £12,000 − £30,000 = £5,000
40% of £5,000 = £2,000
Total tax = £6,000 + £2,000 = £8,000
Answer: £8,000

2 Quadratic inequalities

Solve $3x^2 > 243$

Rearrange: $3x^2 - 243 > 0$
Divide by 3: $x^2 - 81 > 0$
Solve $x^2 - 81 = 0$: $(x + 9)(x - 9) = 0$, so $x = -9$ or $x = 9$
The graph is a U shape crossing the x-axis at -9 and 9; it is above the axis outside the roots
$x < -9$ or $x > 9$
Answer: $x < -9$ or $x > 9$

3 Open box of fixed surface area

An open tray has a square base of side x cm and height h cm. The total outside surface area is 192 cm^2.

(a) Show that the volume of the box is $V = \frac{1}{4}(192x - x^3)$ cm^3.

(b) Find the maximum volume of the box, proving that it is a maximum.

$x^2 + 4xh = 192$, so $h = \frac{192 - x^2}{4x}$

$V = x^2h = \frac{x(192 - x^2)}{4} = \frac{1}{4}(192x - x^3)$

$\frac{dV}{dx} = -\frac{3}{4}x^2 + 48 = 0$, so $x^2 = 64$ and $x = 8$

$\frac{d^2V}{dx^2} = -\frac{3}{2}x = -12 < 0$, so a maximum

$V = 256$
Answer: 256 cm^3

4 Error intervals

The number x is 850 correct to the nearest 50.
Write down the error interval for x.

The rounding unit is 50, so the half-unit is 25
Lower bound = 850 − 25 = 825; upper bound = 850 + 25 = 875
Answer: $825 \leq x < 875$

5 Arc length and sector area

A sector of a circle has radius 5 cm.

The length of the arc of the sector is $\frac{25}{6}\pi$ cm.

(a) Work out the angle of the sector.
(b) Work out the area of the sector. Give your answer to (b) in terms of π.

(a) Arc length = $\frac{\theta}{360} \times 2\pi \times 5$, so $\frac{\theta}{360} \times 10 = \frac{25}{6}$

$\theta = \frac{25}{6} \times \frac{360}{10} = 150°$

(b) Area = $\frac{150}{360} \times \pi \times 5^2 = \frac{125}{12}\pi$ cm^2

Answer: (a) 150° (b) $\frac{125}{12}\pi$ cm^2

6 Cumulative frequency graph: median and quartiles

The cumulative frequency graph shows the masses, in kg, of 120 parcels.
(a) Find the median. (b) Find the interquartile range.
(c) How many parcels have a mass greater than 60?

(a) Median at cumulative frequency 60: reading across gives 50
(b) Lower quartile at 30 gives 40, upper quartile at 90 gives 60
IQR = 60 − 40 = 20
(c) At 60 the cumulative frequency is 90, so 120 − 90 = 30
Answer: (a) 50 (b) 20 (c) 30

7 Equation of a circle

A and B are the points (−4, −2) and (−2, 10).
AB is a diameter of a circle.
Find an equation of the circle.

The centre is the midpoint of AB: $(\frac{-4 + (-2)}{2}, \frac{-2 + 10}{2}) = (-3, 4)$

$r^2 = (-4 - (-3))^2 + (-2 - 4)^2 = 1 + 36 = 37$

$(x + 3)^2 + (y - 4)^2 = 37$

Answer: $(x + 3)^2 + (y - 4)^2 = 37$

8 No stationary points

A curve has equation $y = x^3 + 9x - 5$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 3x^2 + 9 = 3x^2 + 9$

A squared term is never negative, so $\frac{dy}{dx} \geq 9 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 9 (when x = 0)

Answer: (b) 9

9 Estimated mean with an unknown frequency

The table gives information about the times taken by some people to travel to work.
An estimate for the mean is 30.2.
Work out the value of x.

Mid-points: 15, 25, 35, 45, 55
Estimated mean = $\Sigma fm \div \Sigma f$, so $(1195 + 45x) \div (43 + x) = 30.2$
$1195 + 45x = 30.2(43 + x) = 1298.6 + 30.2x$
$(45 - 30.2)x = 1298.6 - 1195$
$14.8x = 103.6$, so $x = 7$
Answer: x = 7

10 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = -t^3 + 6t^2 + t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -3t^2 + 12t + 1$

$a = \frac{dv}{dt} = -6t + 12$

Maximum v when a = 0: $-6t + 12 = 0$, so $t = 2$

$\frac{d^2v}{dt^2} = -6 < 0$, so v is a maximum

$v = 13$

Answer: 13 m/s when t = 2 s