Day 156 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The sum of the interior angles of a polygon is 1800°.
Work out the number of sides of the polygon.

Sum of interior angles = $(n - 2) \times 180°$
$(n - 2) \times 180 = 1800$
$n - 2 = 1800 \div 180 = 10$
$n = 12$
Answer: 12

2 Conditional probability from a table

The table shows information about 83 employees.
(a) An employee who travels to work by car is chosen at random. Work out the probability that this employee is a man.
(b) Two of the 83 employees are chosen at random. Work out the probability that both are men.

(a) 46 employees are in the Car column, and 28 of them are men

Probability = $\frac{14}{23}$

(b) There are 48 men, so P(both) = $\frac{48}{83} \times \frac{47}{82} = \frac{1128}{3403}$

Answer: (a) $\frac{14}{23}$ (b) $\frac{1128}{3403}$

3 Expanding three brackets

Expand and simplify $(x - 3)^2(x - 2)$

Expand the first two brackets: $(x - 3)^2 = x^2 - 6x + 9$
Multiply by $(x - 2)$: $x(x^2 - 6x + 9) - 2(x^2 - 6x + 9)$
$= x^3 - 8x^2 + 21x - 18$
Answer: $x^3 - 8x^2 + 21x - 18$

4 Show that dy/dx factorises

$y = (x - 2)^2(x - 4)$

Show that $\frac{dy}{dx} = (x - 2)(3x - 10)$.

Expand: $y = x^3 - 8x^2 + 20x - 16$
$\frac{dy}{dx} = 3x^2 - 16x + 20$
Factorise: $3x^2 - 16x + 20 = (x - 2)(3x - 10)$
Answer: $\frac{dy}{dx} = (x - 2)(3x - 10)$

5 Speed-time graphs

The graph shows the speed of a train during 21 seconds.
(a) Work out the acceleration during the first 6 seconds.
(b) Work out the average speed over the whole 21 seconds.

(a) Acceleration = change in speed ÷ time = $(12 - 0) \div 6 = 2$ m/s^2
(b) Distance = area under the graph
Trapezium: $\frac{0 + 12}{2} \times 6 = 36$; rectangle: $12 \times 7 = 84$; triangle: $\frac{1}{2} \times 8 \times 12 = 48$
Total = 168 m, so average speed = $168 \div 21 = 8$ m/s
Answer: (a) 2 m/s^2 (b) 8 m/s

6 Increasing or decreasing at a point

$y = 2x^3 + 6x^2 - 3$

At the point where $x = 2$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 + 12x$

When $x = 2$: $\frac{dy}{dx} = 48$

48 is positive, so the curve is increasing

Answer: increasing (gradient 48)

7 Reading a trigonometric graph

The diagram shows the graph of $y = a \cos bx° + c$ for $0 \leq x \leq 360$.

a, b and c are integers.

Find the values of a, b and c.

Greatest value = 2, least value = −6

c = ½(2 + (−6)) = −2 and a = ½(2 − (−6)) = 4

One full cycle takes 180° (2 cycles between 0 and 360), so b = 360 ÷ 180 = 2

Answer: a = 4, b = 2, c = −2

8 Stationary points of a quartic

A curve has equation $y = 2x^4 - 4x^2 - 6$.

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 8x = 8x(x - 1)(x + 1)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 1$ or $x = -1$

$y = -6$ when $x = 0$, and $y = -8$ when $x = \pm 1$

$\frac{d^2y}{dx^2} = 24x^2 - 8$: it is −8 at $x = 0$ (maximum) and 16 at $x = \pm 1$ (minimum)

Answer: (0, −6) maximum; (1, −8) and (−1, −8) minima

9 Surds and fractions

Simplify $\frac{1}{2 + \sqrt{3}} - \frac{1}{2 - \sqrt{3}}$

Give your answer with a rational denominator.

Common denominator: $(2 + \sqrt{3})(2 - \sqrt{3}) = 4 - 3 = 1$

Numerator: $(2 - \sqrt{3}) - (2 + \sqrt{3})$

$= -2\sqrt{3}$

$\frac{-2\sqrt{3}}{1} = -2\sqrt{3}$

Answer: $-2\sqrt{3}$

10 Bounds of a compound expression

p = 10.3 and r = 8.6, each correct to 1 decimal place.

q = 1.22, correct to 2 decimal places.

Work out the lower bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom

Top: LB p − UB q = 10.25 − 1.225 = 9.025

Bottom: UB r = 8.65

9.025 ÷ 8.65 = 1.04

Answer: 1.04