Day 155 **IGCSE 10-a-day: Model Solutions**

1 Inequalities

(a) Solve $5 - 2x \leq 19$
(b) n is an integer. Write down all the values of n that satisfy
$4 \leq 2n + 2 < 10$

(a) Subtract 5: $-2x \leq 14$
Divide by -2 and reverse the inequality sign: $x \geq -7$
(b) Subtract 2: $2 \leq 2n < 8$
Divide by 2: $1 \leq n < 4$
Integers: 1, 2, 3
Answer: (a) $x \geq -7$ (b) 1, 2, 3

2 Mean of a changing set

The mean of 11 numbers is 83.
One number is removed and the mean of the remaining 10 numbers is 90.3.
Work out the number that was removed.

Total of the 11 numbers = $11 \times 83 = 913$
Total of the 10 remaining = $10 \times 90.3 = 903$
Number removed = $913 - 903 = 10$
Answer: 10

3 Expand then differentiate

$y = x(x - 4)(2x + 4)$
Find $\frac{dy}{dx}$.

Expand: $y = 2x^3 - 4x^2 - 16x$
$2x^3 \to 6x^2$, $-4x^2 \to -8x$, $-16x \to -16$
$\frac{dy}{dx} = 6x^2 - 8x - 16$
Answer: $\frac{dy}{dx} = 6x^2 - 8x - 16$

4 Rationalising the denominator

Rationalise the denominator of $\frac{29}{3\sqrt{5}}$
Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{5}$:
$\frac{29}{3\sqrt{5}} = \frac{29\sqrt{5}}{15}$
Answer: $\frac{29\sqrt{5}}{15}$

5 Transformation by a matrix: image and description

$M = \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$.
(a) Find the coordinates of the image of the point (4, 3) under the transformation represented by M.
(b) Describe fully the single transformation represented by M.

(a) $\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 4 \\ 3 \end{pmatrix} = \begin{pmatrix} -3 \\ 4 \end{pmatrix}$
The image is $(-3, 4)$
(b) The first column is where (1, 0) goes and the second column is where (0, 1) goes
M represents a rotation of 90° anticlockwise about the origin
Answer: (a) $(-3, 4)$ (b) Rotation of 90° anticlockwise about the origin

6 Stationary points and their nature

$y = 2x^3 + 12x^2 + 18x$

(a) Find the coordinates of the stationary points.

(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 + 24x + 18 = 6(x + 3)(x + 1)$

$\frac{dy}{dx} = 0$ gives $x = -3$ or $x = -1$, so the points are $(-3, 0)$ and $(-1, -8)$

$\frac{d^2y}{dx^2} = 12x + 24$

$x = -3$: $\frac{d^2y}{dx^2} = -12$, which is negative, so a maximum

$x = -1$: $\frac{d^2y}{dx^2} = 12$, which is positive, so a minimum

Answer: (a) (−3, 0) and (−1, −8) (b) (−3, 0) is a maximum, (−1, −8) is a minimum

7 Ratio and algebra

In an animal shelter, the ratio of the number of cats to the number of dogs is 2 : 3.
Then 18 more cats are added and 3 more dogs are added.
The ratio is now 2 : 1.
How many dogs were there at the start?

Let the numbers at the start be 2k and 3k
(2k + 18) : (3k + 3) = 2 : 1, so 1(2k + 18) = 2(3k + 3)
2k + 18 = 6k + 6
−4k = −12, so k = 3
Number of dogs at the start: 3 × 3 = 9
Answer: 9

8 Vector proof: collinear points

OA = **a** and OB = **b**.
OP = **a**, OQ = −**a** − 8**b** and OR = −5**a** − 24**b**.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = −2**a** − 8**b**
QR = OR − OQ = −4**a** − 16**b**
QR = 2 PQ, so PQ and QR are parallel, and they share the point Q
So P, Q and R lie on a straight line
(b) PQ : QR = 2 : 4 = 1 : 2
Answer: (b) 1 : 2

9 Wire cut in two

A piece of wire 136 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is twice its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{(136 - 4x)^2}{18}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $136 - 4x = 6w$, so $w = \frac{136 - 4x}{6}$ and its area is $2w^2 = \frac{(136 - 4x)^2}{18}$

$A = x^2 + \frac{(136 - 4x)^2}{18} = \frac{17}{9}x^2 - \frac{544}{9}x + \frac{9248}{9}$

$\frac{dA}{dx} = \frac{34}{9}x - \frac{544}{9} = 0$, so $x = 16$. $\frac{d^2A}{dx^2} = \frac{34}{9} > 0$, so a minimum

Square: 256, rectangle: 24 × 12 = 288, so A = 544
Answer: x = 16, least area 544 cm^2

10 Range and inverse of a quadratic

$g(x) = x^2 + 4x - 2$ for $x \leq -2$.

(a) Find the range of g.

(b) Find $g^{-1}(x)$.

(a) $g(x) = (x + 2)^2 - 6$

$x \leq -2$ includes the minimum point, so $g(x) \geq -6$

(b) $y = (x + 2)^2 - 6$, so $(x + 2)^2 = y + 6$

$x \leq -2$ means $x + 2$ is negative or zero, so $x = -2 - \sqrt{y + 6}$

$g^{-1}(x) = -2 - \sqrt{x + 6}$

Answer: (a) $g(x) \geq -6$ (b) $g^{-1}(x) = -2 - \sqrt{x + 6}$