Day 154 **IGCSE 10-a-day: Model Solutions**

1 Differentiating polynomials

$y = x^4 - 4x + 8$

Find $\frac{dy}{dx}$

Differentiate each term: $x^n \to nx^{n-1}$ and a constant $\to 0$

$x^4 \to 4x^3$, $-4x \to -4$

$\frac{dy}{dx} = 4x^3 - 4$

Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{4x^3 - 4}$

2 Enlargement

Triangle T has vertices A(2, 1), B(2, −11), C(8, 1).

T is enlarged with scale factor $-\frac{3}{2}$ and centre (2, −3).

(a) Write down the coordinates of the image of A.
(b) The area of T is 24 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (2 − 2, 1 − (−3)) = (0, 4)

Multiply by $-\frac{3}{2}$: (0, −6)

Image of A = centre + this vector = (2, −9)

(b) Area scale factor = $(-\frac{3}{2})^2 = \frac{9}{4}$

Area = $24 \times \frac{9}{4}$ = 54 square units

Answer: (a) (2, −9) (b) 54

3 Divide then differentiate

$y = \frac{6x^5 - x^3 - 2x}{x^3}$

Find $\frac{dy}{dx}$.

Divide each term by x^3: $y = 6x^2 - 1 - 2x^{-2}$

$6x^2 \to 12x$, $-2x^{-2} \to 4x^{-3}$

$\frac{dy}{dx} = 12x + 4x^{-3}$

Answer: $\frac{\mathbf{dy}}{\mathbf{dx}} = \mathbf{12x} + \frac{\mathbf{4}}{\mathbf{x^3}}$

4 Exchange rates

Rhys changes £800 into euros at £1 = €1.18.
Rhys spends €190 and then changes the rest back into pounds at £1 = €1.25.
How many pounds does Rhys get back?

£800 × 1.18 = €944
944 − 190 = €754
754 ÷ 1.25 = £603.20
Answer: £603.20

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5 Fractions of fractions

Lena read $\frac{7}{8}$ of a book on Saturday.

On Sunday she read $\frac{3}{5}$ of the pages that were left.

She still had 28 pages to read.
How many pages are in the book?

After the first part, $\frac{1}{8}$ remains

After the second, $\frac{2}{5}$ of $\frac{1}{8}$ remains = $\frac{1}{20}$ of the total

$\frac{1}{20}$ of the total is 28

Total = $28 \div \frac{1}{20} = 560$

Answer: 560 pages

6 Solving trigonometric equations

Solve $\sin x = 0.4$ for $0 \leq x \leq 360$.
Give your answers correct to 1 decimal place. (Calculator)

$\sin x = 0.4$

Principal value: $x = \sin^{-1}(0.4) = 23.6°$
Second solution: $x = 180° - $ (principal value)
All solutions in the range: 23.6, 156.4
Answer: x = 23.6, 156.4

7 Given a stationary point, find k

The curve $y = x^3 + kx$ has a stationary point at $x = -2$.
(a) Find the value of k.
(b) Find the coordinates of the other stationary point.

$\frac{dy}{dx} = 3x^2 + k$

$x = -2$: $12 + k = 0$, so $k = -12$

$\frac{dy}{dx} = 3x^2 - 12 = 3(x + 2)(x - 2)$

Other root: $x = 2$
$y = -16$
Answer: (a) k = −12 (b) (2, −16)

8 Growth and decay

A rare car is worth £10,000.
Its value increases by 3% each year.
After how many whole years will it first be worth more than £17,500? (Calculator)

Multiplier each year = 1.03

After 18 years: $£10{,}000 \times 1.03^{18} = £17{,}024.33$

After 19 years: $£10{,}000 \times 1.03^{19} = £17{,}535.06$

So the first time it is more than £17,500 is after 19 years
Answer: 19 years

9 Quadrilateral: cosine rule, then area

In quadrilateral ABCD, AB = 8 cm, BC = 8 cm, CD = 13 cm,
angle ABC = 119° and angle ACD = 63°.
Work out the area of ABCD. Give your answer correct to 3 significant figures. (Calculator)

$AC^2 = 8^2 + 8^2 - 2 \times 8 \times 8 \times \cos 119°$ so AC = 13.786 cm
Area ABC = ½ × 8 × 8 × sin 119° = 27.99
Area ACD = ½ × 13.786 × 13 × sin 63° = 79.84
Total area = 27.99 + 79.84 = 108

Answer: 108 cm^2

10 Tangency conditions

The line y = x + k is a tangent to the curve $y = 2x^2 + 2x + 5$.
Find the value of k.

Equate: $2x^2 + 2x + 5 = x + k$
Rearrange: $2x^2 + x + 5 - k = 0$
A tangent touches at one point, so $b^2 - 4ac = 0$: $1 - 4 \times 2 \times (5 - k) = 0$
$8k - 39 = 0$
$k = \frac{39}{8}$

Answer: k = $\frac{39}{8}$