Day 153 **IGCSE 10-a-day: Model Solutions**

1 Square roots as powers

$y = -6x^{3/2} - 4x - 6\sqrt{x}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -6x^{3/2} - 4x - 6x^{1/2}$

$-6x^{3/2} \to -9x^{1/2}$, $-4x \to -4$, $-6x^{1/2} \to -3x^{-1/2}$

$\frac{dy}{dx} = -9x^{1/2} - 4 - 3x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = -9x^{1/2} - 4 - 3x^{-1/2}}$

2 Bounds of roots

The volume of a cube is 59 cm^3, correct to the nearest cm^3.
Work out the lower bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$58.5 \leq$ volume < 59.5

Lower bound of edge $= \sqrt[3]{58.5}$

$= 3.88197... = 3.88$ (3 s.f.)

Answer: 3.88 cm

3 Laws of indices with algebra

Simplify $(16x^6y^2)^{\frac{1}{2}}$

Raise each part to the power $\frac{1}{2}$

$16^{\frac{1}{2}} = \sqrt{16} = 4$

$(x^6)^{\frac{1}{2}} = x^3$ and $(y^2)^{\frac{1}{2}} = y^1$

$= 4x^3y$

Answer: $4x^3y$

4 Rates of work

A tank is filled by a tap in 10 minutes.
A drain can empty the full tank in 12 minutes.
The tap and the drain are both open, and the tank is empty to begin with.
How long does it take for the tank to fill?

In 1 minute: the tap fills $\frac{1}{10}$ and the drain empties $\frac{1}{12}$

Net: $\frac{1}{10} - \frac{1}{12} = \frac{2}{120}$

Time $= \frac{120}{2} = 60$ minutes

Answer: 60 minutes

5 Tangent perpendicular to a line

The tangent at the point P on the curve $y = 2x^2 + 4x - 8$ is perpendicular to the line $x - 4y = 8$.
Find the coordinates of P.

The line is $y = \frac{1}{4}x - 2$, so its gradient is $\frac{1}{4}$

The tangent has gradient -4

$\frac{dy}{dx} = 4x + 4 = -4$ so $x = -2$

$y = -8$

Answer: $(-2, -8)$

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6 Exact trigonometry in triangles

ABC is a right-angled triangle.
Work out the exact value of x. Give your answer in its simplest form.
The diagram is not drawn accurately.

x is the hypotenuse AC and the known side is the opposite side BC, so use sin 45°

$\sin 45° = \frac{\sqrt{2}}{2}$

$x = 11 \div \sin 45° = 11 \div \frac{\sqrt{2}}{2}$

$x = 11\sqrt{2}$ (after rationalising the denominator)

Answer: $x = 11\sqrt{2}$

7 Area of a triangle given three sides

Work out the area of triangle ABC.
Give your answer correct to 3 significant figures. (Calculator)

$\cos A = \frac{14^2 + 15^2 - 12^2}{2 \times 14 \times 15} = \frac{277}{420}$

$A = 48.736°$

Area = ½ × 14 × 15 × sin 48.736° = 78.9

Answer: 78.9 cm^2

8 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -2t^3 + 12t^2 + 5t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -6t^2 + 24t + 5$

$a = \frac{dv}{dt} = -12t + 24$

Maximum v when a = 0: $-12t + 24 = 0$, so $t = 2$

$\frac{d^2v}{dt^2} = -12 < 0$, so v is a maximum

$v = 29$

Answer: 29 m/s when t = 2 s

9 Equations with indices

Solve $x^{\frac{2}{3}} = 16$
$x > 0$

Write 16 as a power of 4: $16 = 4^2$

$x^{\frac{2}{3}} = 4^2$, so $x = (4^2)^{\frac{3}{2}}$

$x = 4^3 = 64$

Answer: x = 64

10 Inverse and repeated functions

f(x) = 3x + 9.
(a) Find $f^{-1}(21)$.
(b) Express ff(x) in the form px + q.

(a) Solve 3x + 9 = 21: 3x = 12

$x = 4$ so $f^{-1}(21) = 4$

(b) ff(x) = 3(3x + 9) + 9 = 9x + 36

Answer: (a) 4 (b) ff(x) = 9x + 36