Day 152 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $x^3 - 5x^2 + 5$
(a) Find f′(x).
(b) Work out the value of f′(−3).

(a) $x^3 \rightarrow 3x^2$, $-5x^2 \rightarrow -10x$
f′(x) = $3x^2 - 10x$
(b) f′(−3) = 57
Answer: (a) f′(x) = $3x^2 - 10x$ (b) 57

2 Area of a segment

The circle has centre O and radius 7 cm. A and B are points on the circle and angle AOB = 80°. (Calculator)
Work out the area of the shaded segment. Give your answer correct to 3 significant figures.
The diagram is not drawn accurately.

Area of sector OAB = $\frac{80}{360} \times \pi \times 7^2 = 34.208$ cm^2

Area of triangle OAB = $\frac{1}{2} \times 7 \times 7 \times \sin 80° = 24.128$ cm^2

Segment = sector − triangle = 34.208 − 24.128 = 10.081
= 10.1 cm^2 (3 s.f.)
Answer: 10.1 cm^2

3 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 6 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 12 ÷ 6 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 6 \times 12 + 12 \times 30 + \frac{1}{2} \times 6 \times 12 = 36 + 360 + 36$
= 432
Answer: (a) 2 m/s^2 (b) 432 m

4 Fraction problems

A barrel is $\frac{3}{5}$ full.

62 more litres of oil are added and the barrel is then $\frac{4}{5}$ full.

Work out the capacity of the barrel.

Fraction added = $\frac{4}{5} - \frac{3}{5} = \frac{1}{5}$

$\frac{1}{5}$ of the capacity is 62 litres

Capacity = $62 \div \frac{1}{5} = 310$

Answer: 310 litres

5 Tangent through a given point: find the constant

A curve has equation $y = x^2 + 6x + k$, where k is a constant.
The tangent to the curve at the point where $x = 1$ passes through $(-1, -6)$.
Find the value of k.

$\frac{dy}{dx} = 2x + 6$, so at $x = 1$ the gradient is 8

The point of tangency is $(1, 7 + k)$

Gradient between this point and $(-1, -6)$: $\frac{-6 - (7 + k)}{-1 - 1} = 8$

$-6 - (7 + k) = -16$

$k = 3$

Answer: k = 3

6 Solving AX = B

$A = \begin{pmatrix} 1 & 6 \\ 1 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -19 \\ 9 \end{pmatrix}$.

AX = B, where X is a 2 × 1 matrix.
Find X.

$X = A^{-1}B$ (multiply both sides on the left by A^{-1})

$A^{-1} = -\frac{1}{7}\begin{pmatrix} -1 & -6 \\ -1 & 1 \end{pmatrix}$

$X = -\frac{1}{7}\begin{pmatrix} -1 & -6 \\ -1 & 1 \end{pmatrix}\begin{pmatrix} -19 \\ 9 \end{pmatrix} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

Answer: X = $\begin{pmatrix} \mathbf{5} \\ \mathbf{-4} \end{pmatrix}$

7 Rearranging when the subject appears twice

Make x the subject of $y = \frac{-x + 6}{2x - 1}$

Multiply by $2x - 1$: $y(2x - 1) = -x + 6$
Expand: $2xy - y = -x + 6$
Collect the x terms: $2xy + x = 6 + y$
Factorise: $x(2y + 1) = y + 6$

$x = \frac{y + 6}{2y + 1}$

Answer: x = $\frac{\mathbf{y + 6}}{\mathbf{2y + 1}}$

8 Successive percentage change

A house price increases by 25%.
What percentage decrease would take it back to its original value?

Let the original be 100. After the increase it is 125
Decrease needed = 25
25 ÷ 125 × 100 = 20%

Answer: 20%

9 Open box from a square sheet

Squares of side x cm are cut from the corners of a 24 cm square cardboard sheet, which is folded into an open box of volume V cm^3.

(a) Show that $V = 4x^3 - 96x^2 + 576x$.

(b) Find x for the maximum volume, and this volume.

Base is a square of side $24 - 2x$, so $V = x(24 - 2x)^2 = x(576 - 96x + 4x^2) = 4x^3 - 96x^2 + 576x$

$\frac{dV}{dx} = 12x^2 - 192x + 576 = 0$, so $x^2 - 16x + 48 = 0$

$12(x - 4)(x - 12) = 0$, so $x = 4$ or $x = 12$

$x = 12$ is rejected (12 is more than half of 24). $\frac{d^2V}{dx^2} = 24x - 192$, which is -96 when $x = 4$, so a maximum

$V = 1024$

Answer: x = 4, V = 1024 cm^3

10 Bounds: finding an unknown length

A rectangle has area 96 cm^2, correct to 2 significant figures.

Its length is 14 cm, correct to 2 significant figures.

Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: $95.5 \leq A < 96.5$. Length: $13.5 \leq l < 14.5$

UB width = UB area ÷ LB length = 96.5 ÷ 13.5 = 7.15

Answer: 7.15 cm