Day 151 **IGCSE 10-a-day: Model Solutions**

1 Vector between two points

The position vectors of A and B are $\begin{pmatrix}5\\-3\end{pmatrix}$ and $\begin{pmatrix}6\\7\end{pmatrix}$.

(a) Write down the vector **AB**.

(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix}6\\7\end{pmatrix} - \begin{pmatrix}5\\-3\end{pmatrix} = \begin{pmatrix}1\\10\end{pmatrix}$

(b) $|\mathbf{AB}| = \sqrt{(1)^2 + (10)^2} = \sqrt{101}$

Answer: (a) $\begin{pmatrix}\mathbf{1}\\\mathbf{10}\end{pmatrix}$ **(b)** $\sqrt{\mathbf{101}}$

2 Ordering numbers

Write these numbers in order of size, starting with the smallest.

0.67, 84%, $\frac{10}{17}$, $\frac{3}{4}$

Convert each to a decimal: 0.67 = 0.67, 84% = 0.84, $\frac{10}{17}$ = 0.588, $\frac{3}{4}$ = 0.75

In order: $\frac{10}{17}$, 0.67, $\frac{3}{4}$, 84%

Answer: $\mathbf{\frac{10}{17}}$**, 0.67,** $\mathbf{\frac{3}{4}}$**, 84%**

3 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where

$s = -t^3 + 12t^2 - 36t + 4$ $(t \geq 0)$

Find the distance between the two positions where P is at rest.

$v = \frac{ds}{dt} = -3t^2 + 24t - 36$

P is at rest when $v = 0$: $-3t^2 + 24t - 36 = 0$

$-3(t - 2)(t - 6) = 0$, so $t = 2$ or $t = 6$

$s(2) = -28$ and $s(6) = 4$

Distance $= |4 - (-28)| = 32$

Answer: 32 m

4 Mean of a changing set

The mean of 6 numbers is 152.

One number is removed and the mean of the remaining 5 numbers is 176.8.

Work out the number that was removed.

Total of the 6 numbers $= 6 \times 152 = 912$

Total of the 5 remaining $= 5 \times 176.8 = 884$

Number removed $= 912 - 884 = 28$

Answer: 28

5 Trigonometric equations

Solve $2 \sin x° - 2 = -2$ for $0 \leq x \leq 360$

Rearrange: $2 \sin x° = 0$, so $\sin x° = 0$

From the graph of $y = \sin x°$, $\sin x° = 0$ when $x = 0$ or 180 or 360

Answer: x = 0, 180, 360

6 Recasting solids

A solid cylinder of radius 6 cm and height 8 cm is melted down.
The metal is used to make identical solid spheres of radius 3 cm.
Work out the number of spheres that can be made.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of cylinder = $\pi \times 6^2 \times 8 = 288\pi$

Volume of one sphere = $\frac{4}{3} \times \pi \times 3^3 = 36\pi$

Number of spheres = $288 \div 36 = 8$

Answer: 8

7 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx + 4$ has stationary points at $x = -3$ and $x = 3$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$

$x = -3$: $27 + -6a + b = 0$. $x = 3$: $27 + 6a + b = 0$

Subtract the equations: $0 + -12a = 0$, so $a = 0$

Then $b = -27$

Answer: a = 0, b = −27

8 Angles of elevation from two points

Points A and B are on level ground, 100 m apart, with B between A and the foot of a lighthouse.
The angle of elevation of the top of the lighthouse is 43° from A and 63° from B.
Work out the height of the lighthouse. Give your answer to 3 significant figures. (Calculator)

Let the top be T. Angle ATB = 63° − 43° = 20° (exterior angle of the triangle)

Sine rule in triangle ABT: $\frac{AT}{\sin 117°} = \frac{100}{\sin 20°}$

$AT = \frac{100 \times \sin 63°}{\sin 20°} = 260.51$

Height = AT × sin 43° = 178 m

Answer: 178 m

9 Curves that touch

The curves $y = 2x^2 - 4x - 2$ and $y = -x^2 - 10x - 5$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $3x^2 + 6x + 3 = 0$

That is $3(x + 1)^2 = 0$ with a repeated root $x = -1$

$y = 4$, so the point is $(-1, 4)$

$\frac{dy}{dx} = 4x - 4$ on the first curve: gradient -8

Tangent: $y = -8x - 4$

Answer: (−1, 4), y = −8x − 4

10 Tree diagrams and reverse probability

Machine A makes 40% of the bolts in a factory. Machine B makes the rest.
50% of the bolts from machine A are faulty.
10% of the bolts from machine B are faulty.
(a) Work out the probability that a bolt chosen at random is faulty.
(b) A faulty bolt is found. Work out the probability that it was made by machine A.

(a) P(faulty) = 0.4 × 0.5 + 0.6 × 0.1 = 0.2 + 0.06 = 0.26
(b) P(A and faulty) = 0.4 × 0.5 = 0.2

P(A | faulty) = 0.2 ÷ 0.26 = $\frac{10}{13}$

Answer: (a) 0.26 (b) $\frac{10}{13}$