Day 150 **IGCSE 10-a-day: Model Solutions**

1 Equation of a normal

A curve has equation $y = 3x^2 - 2x - 4$.
Find the equation of the normal to the curve at the point where $x = 1$.

When $x = 1$, $y = -3$ and $\frac{dy}{dx} = 4$

Gradient of the normal $= -1 \div 4 = -\frac{1}{4}$

$y + 3 = -\frac{1}{4}(x - 1)$

$y = -\frac{1}{4}x - \frac{11}{4}$

Answer: $y = -\frac{1}{4}x - \frac{11}{4}$

2 Solving trigonometric equations

Solve $2 \sin x = -\sqrt{3}$ for $0° \leq x \leq 360°$.
Do not use a calculator.

$\sin x = -\frac{\sqrt{3}}{2}$

The related acute angle satisfies $\sin \theta = \frac{\sqrt{3}}{2}$, so $\theta = 60°$

sine is negative in the 3rd and 4th quadrants, so $x = 180° + 60°$ and $x = 360° - 60°$
$x = 240°$ and $x = 300°$
Answer: $x = 240°$ or $x = 300°$

3 Expand then differentiate

$y = (x - 1)(x + 2)$
Find $\frac{dy}{dx}$.

Expand: $y = x^2 + x - 2$
$x^2 \rightarrow 2x$, $x \rightarrow 1$
$\frac{dy}{dx} = 2x + 1$

Answer: $\frac{dy}{dx} = 2x + 1$

4 Inverse of a linear function

$f(x) = \frac{4x - 5}{2}$.

(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(1)$.

(a) Let $y = \frac{4x - 5}{2}$, then rearrange to make x the subject

$x = \frac{2y + 5}{4}$, so $f^{-1}(x) = \frac{2x + 5}{4}$

(b) $f^{-1}(1) = \frac{7}{4}$

Answer: (a) $f^{-1}(x) = \frac{2x + 5}{4}$ (b) $\frac{7}{4}$

5 Mean of a changing set

The mean of 7 numbers is 44.
A new number is added and the mean of all 8 numbers is 42.5.
Work out the new number.

Total of the 7 numbers = $7 \times 44 = 308$
New total = $8 \times 42.5 = 340$
New number = $340 - 308 = 32$
Answer: 32

6 Venn diagram with algebra

The Venn diagram shows information about 27 families.
A: have a garden B: have a car
(a) Find the value of x.
(b) Find n(A) and n(B ∩ A′).

(a) $6x + 3 = 27$
$6x = 24$, so $x = 4$
(b) $n(A) = 3x + 2 + x = 14 + 4 = 18$
$n(B \cap A') = 2x - 1 = 7$
Answer: (a) x = 4 (b) n(A) = 18, n(B ∩ A′) = 7

7 Tangent to a factorised cubic

A curve has equation $y = (x + 1)(x - 3)(x + 2)$.
Find the equation of the tangent to the curve at the point where $x = -2$.

Expand: $y = x^3 - 7x - 6$
$\frac{dy}{dx} = 3x^2 - 7$
At $x = -2$: $y = 0$ and gradient = 5
$y = 5(x + 2)$
$y = 5x + 10$
Answer: y = 5x + 10

8 Area and quadratic equations

The diagram shows a rectangle of length $(3x - 2)$ cm, width $(2x + 1)$ cm and area 374 cm^2.
(a) Show that $6x^2 - x - 376 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(3x - 2)(2x + 1) = 374$
Expand: $6x^2 - x - 2 = 374$, so $6x^2 - x - 376 = 0$
(b) Factorise: $(x - 8)(6x + 47) = 0$
$x = 8$ ($x = -\frac{47}{6}$ is not possible as a length would be negative)
Length = 22 cm and width = 17 cm
Perimeter = $2 \times (22 + 17) = 78$ cm
Answer: (b) 78 cm

9 Counting factors

$N = 2 \times 3 \times 7^4$
How many factors does N have?
Include 1 and N.

Add 1 to each power and multiply
$(1 + 1) \times (1 + 1) \times (4 + 1) = 2 \times 2 \times 5 = 20$
Answer: 20

10 Direct and inverse proportion

y is directly proportional to the cube of x.
When x = 5, y = 1250.
(a) Find a formula for y in terms of x.
(b) Find the value of x when y = 80.

(a) $y = kx^3$

Substitute x = 5 and y = 1250: $1250 = k \times 5^3$

$k = 10$

$y = 10x^3$

(b) $80 = 10x^3$, so $x^3 = 8$, so $x = 2$

Answer: (a) $y = 10x^3$ (b) $x = 2$