**Day 15** **IGCSE 10-a-day: Model Solutions**

## 1 Tangent at a turning point

The graph of $y = 3x^2 + 12x + 7$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 6x + 12$

At P, $\frac{dy}{dx} = 0$, so $6x + 12 = 0$ and $x = -2$

$y = -5$ at P
The gradient is 0, so the tangent is horizontal: $y = -5$
**Answer: $y = -5$**

## 2 Equations with fractions

Solve $\frac{x + 7}{2} + \frac{x + 4}{8} = 9$

Multiply every term by 8: $4(x + 7) + (x + 4) = 72$
Expand: $5x + 32 = 72$
$5x = 40$
$x = 8$
**Answer: $x = 8$**

## 3 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by
$v = 18t - 3t^2$ for $0 \leq t \leq 6$
Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -6t + 18$

Greatest speed when $a = 0$: $-6t + 18 = 0$, so $t = 3$

$\frac{d^2v}{dt^2} = -6 < 0$, so this is a maximum

$v = 18 \times 3 - 3 \times 3^2 = 27$
**Answer: t = 3 s, greatest speed 27 m/s**

## 4 Arithmetic sequence from two terms

An arithmetic sequence has term 4 equal to 1 and term 10 equal to −53.
(a) Find an expression for the nth term.
(b) Find the first term of the sequence that is less than −100.

(a) Common difference $= (-53 - 1) \div 6 = -9$
nth term $= -9n + c$; $1 = (-9) \times 4 + c$ so $c = 37$
nth term $= -9n + 37$

(b) $-9n + 37 < -100$: $n < \frac{137}{9}$

Smallest whole number $n = 16$; the term is $-107$
**Answer: (a) $-9n + 37$ (b) $-107$**

## 5 Distance-time graphs

The graph shows Carlos's cycle ride to a café and back.
(a) Work out Carlos's speed on the way back.
(b) Work out Carlos's average speed for the whole ride, including the stop.

(a) Return: 10 km in 0.5 hours, so speed $= 10 \div 0.5 = 20$ km/h
(b) Total distance $= 2 \times 10 = 20$ km, total time = 2 hours
Average speed $= 20 \div 2 = 10$ km/h
**Answer: (a) 20 km/h (b) 10 km/h**

## 6 Points satisfying inequalities

R is the region that satisfies all of these inequalities:
$y < 2x$ $x + y > 10$ $x > 1$
Which of these points lie in R?
A(1, 1) B(3, 3) C(4, 0) D(8, 4) E(8, 7)

A(1, 1): $y < 2$: yes; $x + y = 2$: no; $x = 1$: no
B(3, 3): $y < 6$: yes; $x + y = 6$: no; $x = 3$: yes
C(4, 0): $y < 8$: yes; $x + y = 4$: no; $x = 4$: yes
D(8, 4): $y < 16$: yes; $x + y = 12$: yes; $x = 8$: yes
E(8, 7): $y < 16$: yes; $x + y = 15$: yes; $x = 8$: yes
In R: D, E
**Answer: D, E**

## 7 Algebraic fractions: adding and subtracting

Write $\frac{1}{x-6} + \frac{x+2}{x-3}$ as a single fraction.
Simplify the numerator.

The common denominator is $(x - 6)(x - 3)$

$$\frac{1}{x-6} + \frac{x+2}{x-3} = \frac{(x-3) + (x+2)(x-6)}{(x-6)(x-3)}$$

Expand the numerator: $x^2 - 3x - 15$

$$\frac{x^2 - 3x - 15}{(x-6)(x-3)}$$

**Answer:** $\mathbf{\frac{x^2 - 3x - 15}{(x-6)(x-3)}}$

## 8 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 4^2 + 7^2 = 65$
$AG^2 = AC^2 + CG^2 = 65 + 4^2 = 81$
$AG = 9$ cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{65}}$

angle GAC = 26.4° (1 d.p.)
**Answer: (a) 9 cm (b) 26.4°**

## 9 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - 3x^2 + 6x + 3$ at the point P where $x = 2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 6x + 6$, so at P the gradient is 6 and $y = 11$
Tangent: $y = 6x - 1$
Intersection: $x^3 - 3x^2 + 4 = 0$
$x = 2$ is a repeated root, so the roots are 2, 2 and $x_Q$
Sum of roots = 3, so $4 + x_Q = 3$ and $x_Q = -1$
$y = -7$
**Answer: (−1, −7)**

## 10 Counting factors

$N = 2^2 \times 3^3 \times 7^2$

How many factors does N have?

Include 1 and N.

Add 1 to each power and multiply

$(2 + 1) \times (3 + 1) \times (2 + 1) = 3 \times 4 \times 3 = 36$

**Answer: 36**