Day 149 **IGCSE 10-a-day: Model Solutions**

1 Gradient at a point

Work out the gradient of the curve $y = -2x^4 - 3x + 6$ at the point where $x = 3$.

$\frac{dy}{dx} = -8x^3 - 3$

When $x = 3$: gradient $= -219$

Answer: -219

2 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 9 cm, DB = 3 cm and the area of triangle ADE is 162 cm^2.
Work out the area of the trapezium DBCE.
The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 9 + 3 = 12 cm

Length scale factor = $12 \div 9 = \frac{4}{3}$

Area scale factor = $(\frac{4}{3})^2 = \frac{16}{9}$

Area of triangle ABC = $162 \times \frac{16}{9} = 288$ cm^2

Trapezium = $288 - 162 = 126$ cm^2

Answer: 126 cm^2

3 Forming and solving quadratic equations

A rectangle has length (x + 6) cm and width (x + 3) cm.
The area of the rectangle is 70 cm^2.
(a) Show that $x^2 + 9x - 52 = 0$
(b) Work out the perimeter of the rectangle.

(a) $(x + 6)(x + 3) = 70$, so $x^2 + 9x + 18 = 70$

$x^2 + 9x - 52 = 0$

(b) $x^2 + 9x - 52 = 0$: $(x - 4)(x + 13) = 0$

$x = 4$ ($x = -13$ is not possible as lengths are positive)

Length = 10 cm and width = 7 cm, so perimeter $= 2 \times (10 + 7) = 34$ cm

Answer: (b) 34 cm

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = $(2x + 20)°$, angle BCD = $(5x - 36)°$ and angle ABC = $(3x - 30)°$.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°

$(2x + 20) + (5x - 36) = 180$

$7x - 16 = 180$, so $7x = 196$

$x = 28$

(b) Angle ABC $= 3 \times 28 - 30 = 54°$

Angle ADC $= 180 - 54 = 126°$

Answer: (a) x = 28 (b) 126°

5 Square roots as powers

$y = 8x^{3/2} - 2\sqrt{x}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = 8x^{3/2} - 2x^{1/2}$

$8x^{3/2} \to 12x^{1/2}$, $-2x^{1/2} \to -x^{-1/2}$

$\frac{dy}{dx} = 12x^{1/2} - x^{-1/2}$

Answer: $\mathbf{\frac{dy}{dx} = 12x^{1/2} - x^{-1/2}}$

6 Inverse variation

y is inversely proportional to the square root of x.
When x = 25, y = 14.4.
(a) Find a formula for y in terms of x.
(b) Find y when x = 9.

$y = \frac{k}{\sqrt{x}}$

$14.4 = \frac{k}{\sqrt{25}}$ so $k = 14.4 \times 5 = 72$

$y = \frac{72}{\sqrt{x}}$

$y = \frac{72}{3} = 24$

Answer: (a) $\mathbf{y = \frac{72}{\sqrt{x}}}$ **(b) y = 24**

7 Recurring decimals

Without using a calculator, work out $0.\dot{4} \times 0.\dot{1}\dot{3}$
Give your answer as a fraction in its simplest form.

$0.\dot{4} = \frac{4}{9}$

$0.\dot{1}\dot{3} = \frac{13}{99}$

$\frac{4}{9} \times \frac{13}{99} = \frac{52}{891}$

Answer: $\mathbf{\frac{52}{891}}$

8 Show that dy/dx = ...

$y = \frac{(2x + 2)(3x - 2)}{x}$ $(x \neq 0)$

(a) Show that $\frac{dy}{dx} = \frac{6x^2 + 4}{x^2}$.

(b) Work out the gradient of the curve when $x = -1$.

Expand the brackets: $(2x + 2)(3x - 2) = 6x^2 + 2x - 4$, then divide each term by x: $y = 6x + 2 - 4x^{-1}$

$\frac{dy}{dx} = 6 + \frac{4}{x^2}$

$= \frac{6x^2 + 4}{x^2}$

When $x = -1$: $\frac{dy}{dx} = 10$

Answer: (b) 10

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to t.
When v = 2 and t = 1, P = 20.
Work out the value of P when v = 4 and t = 5.

$P = \frac{kv^2}{t}$

$20 = \frac{k \times 4}{1}$, so $k = 20 \times 1 \div 4 = 5$

$P = \frac{5 \times 16}{5} = 16$

Answer: P = 16

10 Exact values from a given ratio

x is an acute angle and $\sin x = \frac{4}{7}$.

Find the exact value of (a) cos x (b) tan x.
Give each answer as a fraction, rationalising any denominator.

Right-angled triangle: opposite = 4, hypotenuse = 7

Adjacent = $\sqrt{7^2 - 4^2} = \sqrt{33}$

(a) $\cos x = \frac{\text{adjacent}}{\text{hypotenuse}} = \frac{\sqrt{33}}{7}$

(b) $\tan x = \frac{4}{\sqrt{33}} = \frac{4\sqrt{33}}{33}$

Answer: (a) $\frac{\sqrt{33}}{7}$ (b) $\frac{4\sqrt{33}}{33}$