Day 148 **IGCSE 10-a-day: Model Solutions**

1 Vector between two points

A is the point $(-2, 0)$ and B is the point $(-5, -6)$.
(a) Write down the vector $\mathbf{AB}$.
(b) Find $|\mathbf{AB}|$, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix} -5 \\ -6 \end{pmatrix} - \begin{pmatrix} -2 \\ 0 \end{pmatrix} = \begin{pmatrix} -3 \\ -6 \end{pmatrix}$

(b) $|\mathbf{AB}| = \sqrt{(-3)^2 + (-6)^2} = \sqrt{45} = 3\sqrt{5}$

Answer: (a) $\begin{pmatrix} \mathbf{-3} \\ \mathbf{-6} \end{pmatrix}$ **(b)** $\mathbf{3\sqrt{5}}$

2 Equations with fractions

Solve $\frac{x-5}{6} + \frac{x+3}{2} = 4$

Multiply every term by 6: $(x - 5) + 3(x + 3) = 24$
Expand: $4x + 4 = 24$
$4x = 20$
$x = 5$
Answer: x = 5

3 Expand then differentiate

$y = (x + 2)(x - 5)$
Find $\frac{dy}{dx}$.

Expand: $y = x^2 - 3x - 10$
$x^2 \rightarrow 2x$, $-3x \rightarrow -3$
$\frac{dy}{dx} = 2x - 3$
Answer: $\mathbf{\frac{dy}{dx} = 2x - 3}$

4 nth term of a quadratic sequence

Here are the first five terms of a quadratic sequence.
20, 39, 66, 101, 144
Find an expression for the nth term.

First differences: 19, 27, 35, 43. Second differences: 8, 8, 8
Second difference = 2a, so a = 4
Subtract $4n^2$: 16, 23, 30, 37, 44
This is linear: 7n + 9
nth term = $4n^2 + 7n + 9$
Answer: $\mathbf{4n^2 + 7n + 9}$

5 Nested surds

Show that $\sqrt{43 + 24\sqrt{3}} = 4 + 3\sqrt{3}$

Square the right-hand side: $(4 + 3\sqrt{3})^2 = 16 + 24\sqrt{3} + 27$
$= 43 + 24\sqrt{3}$
So $\sqrt{43 + 24\sqrt{3}} = 4 + 3\sqrt{3}$
Answer: $\mathbf{\sqrt{43 + 24\sqrt{3}} = 4 + 3\sqrt{3}}$

6 Stationary point of a quartic

$y = -x^4 + 4x$

Find the coordinates of the stationary point and determine its nature.

$\frac{dy}{dx} = -4x^3 + 4$

$\frac{dy}{dx} = 0$: $-4x^3 + 4 = 0$, so $x^3 = 1$

$x = 1$ and $y = 3$

$\frac{d^2y}{dx^2} = -12x^2$

At $x = 1$: $\frac{d^2y}{dx^2} = -12$, so a maximum

Answer: (1, 3), maximum

7 Geometric sequences

The 2nd term of a geometric sequence is 3.
The 5th term of the sequence is 81.
(a) Work out the common ratio.
(b) Work out the first term.

(a) The common ratio r satisfies $r^3 = 81 \div 3 = 27$

$r = \sqrt[3]{27} = 3$

(b) The 2nd term is $a \times r^1$: $a \times 3 = 3$

$a = 3 \div 3 = 1$

Answer: (a) r = 3 (b) 1

8 Increasing range for y = ax + b/x

$y = 5x + \frac{80}{x}$

Find the values of x for which y is increasing.

$\frac{dy}{dx} = 5 - \frac{80}{x^2}$

$\frac{dy}{dx} > 0$: $5 > \frac{80}{x^2}$

$x^2 > 16$ (x^2 is positive, so multiply through)

$x < -4$ or $x > 4$

Answer: $x < -4$ or $x > 4$

9 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 6 cm, top radius 2 cm and base radius 5 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+6} = \frac{2}{5}$

$5h = 2(h + 6)$, so $3h = 12$ and $h = 4$

Large cone: $\frac{1}{3} \times \pi \times 5^2 \times 10 = \frac{250}{3}\pi$

Small cone: $\frac{1}{3} \times \pi \times 2^2 \times 4 = \frac{16}{3}\pi$

Frustum = $\frac{250}{3}\pi - \frac{16}{3}\pi = 78\pi$ cm^3

Answer: 78π cm^3

10 Joint and inverse variation

y is directly proportional to x and inversely proportional to z.
When x = 5 and z = 4, y = 15.
Work out the value of y when x = 1 and z = 1.

$y = \frac{kx}{z}$

$15 = \frac{k \times 5}{4}$, so $k = 15 \times 4 \div 5 = 12$

$y = \frac{12 \times 1}{1} = 12$

Answer: y = 12