Day 147 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $a^2 - 3bc$
when $a = -4$, $b = 4$ and $c = 2$.

Substitute the values: $(-4)^2 - 3 \times 4 \times 2$
$= 16 - 24$
$= -8$

Answer: -8

2 With and without replacement

A bag has 2 black and 7 white counters.
Two counters are taken at random. Work out the probability that both are black when
(a) the first is replaced before the second is taken
(b) the first is not replaced

(a) $\frac{2}{9} \times \frac{2}{9} = \frac{4}{81}$

(b) After taking one black there are 1 black and 8 in total
$\frac{2}{9} \times \frac{1}{8} = \frac{1}{36}$

Answer: (a) $\frac{4}{81}$ (b) $\frac{1}{36}$

3 Fraction problems

A bag contains balloons.
$\frac{1}{5}$ of the balloons are pink and $\frac{2}{5}$ of the balloons are silver.
The rest of the balloons, 48 of them, are gold.
How many pink balloons are there?

Fraction that are gold $= 1 - \frac{1}{5} - \frac{2}{5} = \frac{2}{5}$

$\frac{2}{5}$ of the total is 48

Total $= 48 \div \frac{2}{5} = 120$

Pink balloons $= \frac{1}{5}$ of $120 = 24$

Answer: 24

4 Gradient from a tangent

The diagram shows part of the curve $y = x^2 + 4x - 1$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

Gradient $= \frac{3 - (-5)}{1 - (-1)} = 4$

$\frac{dy}{dx} = 2x + 4$

$2x + 4 = 4$, so $x = 0$

Answer: (a) 4 (b) $x = 0$

5 Averages with algebra

Here are five numbers: $3x - 3$, $x - 2$, x, $2x + 1$, $x + 1$.
The mean of the five numbers is 9.
(a) Work out the value of x.
(b) Work out the median of the five numbers.

(a) Total = $(3x - 3) + (x - 2) + (x) + (2x + 1) + (x + 1) = 9 \times 5 = 45$
$8x - 3 = 45$, so $8x = 48$ and $x = 6$
The numbers are 15, 4, 6, 13, 7
(b) In order: 4, 6, 7, 13, 15, so the median is 7
Answer: (a) x = 6 (b) 7

6 Compound interest: find the rate

£8,000 is invested for 3 years at r% per year compound interest.
At the end of 3 years the investment is worth £8,741.82.
Work out the value of r. (Calculator)

$8000 \times (1 + \frac{r}{100})^3 = £8{,}741.82$

$(1 + \frac{r}{100})^3 = 8741.82 \div 8000 = 1.0927275$

$1 + \frac{r}{100} = 1.03$

$r = 3$

Answer: r = 3

7 Trigonometric graphs

The diagram shows the graph of $y = a \cos bx$, where a and b are positive integers.
(a) Write down the value of a and the value of b.
(b) Work out the value of y when $x = 105°$.

(a) The maximum value is 3 and the minimum is -3, so $a = 3$
The graph completes 4 full cycles between 0° and 360°, so $b = 4$
(b) $y = 3 \cos (4 \times 105°) = 3 \cos 420°$

$\cos 60° = \frac{1}{2}$, so $y = \frac{3}{2}$

Answer: (a) a = 3, b = 4 (b) y = $\frac{3}{2}$

8 Stationary points of a quartic

A curve has equation $y = 2x^4 - 36x^2 + 5$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 72x = 8x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 3$ or $x = -3$

$y = 5$ when $x = 0$, and $y = -157$ when $x = \pm 3$

$\frac{d^2y}{dx^2} = 24x^2 - 72$: it is -72 at $x = 0$ (maximum) and 144 at $x = \pm 3$ (minimum)

Answer: (0, 5) maximum; (3, −157) and (−3, −157) minima

9 Iteration

Use the iteration formula $x_{n+1} = \sqrt[3]{5x_n + 1}$ with $x_0 = 2$ (Calculator)
to work out x_1, x_2 and x_3.
Give each answer correct to 3 decimal places.

$x_1 = \sqrt[3]{5 \times 2 + 1} = 2.22398...$

$x_2 = \sqrt[3]{5 \times x_1 + 1} = 2.29703...$

$x_3 = \sqrt[3]{5 \times x_2 + 1} = 2.31987...$

Rounded: $x_1 = 2.224$, $x_2 = 2.297$, $x_3 = 2.320$

Answer: $x_1 = 2.224$, $x_2 = 2.297$, $x_3 = 2.320$

10 Distance travelled

The diagram shows the graph of $s = t^3 - 9t^2 + 15t$, the displacement in metres of a particle P from O after t seconds.
P changes direction at A and B. Work out the total distance travelled in the first 7 seconds.

$v = \frac{ds}{dt} = 3t^2 - 18t + 15 = 0$ at $t = 1$ and $t = 5$ (A and B)

$s(0) = 0$, $s(1) = 7$, $s(5) = -25$, $s(7) = 7$

Distances: $|7 - 0| + |-25 - 7| + |7 - (-25)|$

$= 7 + 32 + 32 = 71$

Answer: 71 m