Day 146 **IGCSE 10-a-day: Model Solutions**

1 Parallelogram and midpoint with vectors

ABCD is a parallelogram.
A = (−2, 2), B = (3, −1) and D = (2, 4).
Find the coordinates of C and of the point where the diagonals meet.

$\mathbf{BC} = \mathbf{AD} = \text{D} - \text{A} = \begin{pmatrix} 4 \\ 2 \end{pmatrix}$

C = B + **AD** = (7, 1)

The diagonals bisect each other, so they meet at the midpoint of AC

$(\frac{-2+7}{2}, \frac{2+1}{2}) = (\frac{5}{2}, \frac{3}{2})$

Answer: C = (7, 1), diagonals meet at $(\frac{5}{2}, \frac{3}{2})$

2 Reverse percentages

The price of a watch is increased by 20%.
Later the new price is reduced by 15% in a sale.
The sale price is £40.80.
Work out the price before the increase. (Calculator)

Multipliers: 1.2 and 0.85
Overall multiplier = 1.2 × 0.85 = 1.02
Original × 1.02 = £40.80
Original = £40.80 ÷ 1.02 = £40
Answer: £40

3 Fractions

Work out $2\frac{1}{3} \div 2\frac{2}{5} + 3\frac{1}{4}$

Give your answer as a mixed number or a fraction in its simplest form.

Change the mixed numbers to improper fractions

$\frac{7}{3} \div \frac{12}{5} = \frac{7}{3} \times \frac{5}{12} = \frac{35}{36}$

$\frac{35}{36} + \frac{13}{4} = \frac{38}{9}$

$= 4\frac{2}{9}$

Answer: $4\frac{2}{9}$

4 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 48°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 48°
Angle COB is an exterior angle of triangle OAC: 48 + 48 = 96°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 96°

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5 Iteration: rearranging equations

(a) Show that the equation $x^3 - 5x - 34 = 0$ has a solution between x = 3 and x = 4.

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{5x + 34}$

(a) Let $f(x) = x^3 - 5x - 34$. $f(3) = 3^3 - 5 \times 3 - 34 = -22$ and $f(4) = 4^3 - 5 \times 4 - 34 = 10$

f(3) is negative and f(4) is positive. There is a change of sign, so there is a solution between 3 and 4

(b) Rearrange $x^3 - 5x - 34 = 0$: $x^3 = 5x + 34$

Take the cube root of both sides: $x = \sqrt[3]{5x + 34}$

Answer: Shown

6 Minimum point of y = ax + b/x

The diagram shows a sketch of the curve $y = 2x + \frac{2}{x}$ for x > 0.

The curve has a minimum point at M.

Find the coordinates of M, justifying that it is a minimum.

$y = 2x + 2x^{-1}$, so $\frac{dy}{dx} = 2 - 2x^{-2}$

$\frac{dy}{dx} = 0$: $2 = \frac{2}{x^2}$, so $x^2 = 1$ and x = 1 (x > 0)

$\frac{d^2y}{dx^2} = 4x^{-3} = 4 > 0$, so a minimum

$y = 2 + \frac{2}{1} = 4$

Answer: M (1, 4)

7 Rationalising the denominator

Write $\frac{8}{4 + \sqrt{10}}$ in the form $\frac{p + q\sqrt{10}}{r}$

where p, q and r are integers and r > 1,

and the fraction is in its simplest form.

Multiply top and bottom by $(4 - \sqrt{10})$:

$\frac{8}{4 + \sqrt{10}} = \frac{8(4 - \sqrt{10})}{(4 + \sqrt{10})(4 - \sqrt{10})}$

Denominator: 16 − 10 = 6

$= \frac{16 - 4\sqrt{10}}{3}$

Answer: $\frac{16 - 4\sqrt{10}}{3}$

8 Gradient from a tangent

The diagram shows part of the curve $y = -x^2 - 4x + 2$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)

(a) Use the two marked points on the tangent to work out its gradient.

(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{-2 - 6}{1 - (-1)} = -4$

$\frac{dy}{dx} = -2x - 4$

$-2x - 4 = -4$, so x = 0

Answer: (a) −4 (b) x = 0

9 Range and inverse of a quadratic

$g(x) = x^2 + 2x + 8$ for $x \geq -1$.
(a) Find the range of g.
(b) Find $g^{-1}(x)$.

(a) $g(x) = (x + 1)^2 + 7$
$x \geq -1$ includes the minimum point, so $g(x) \geq 7$
(b) $y = (x + 1)^2 + 7$, so $(x + 1)^2 = y - 7$
$x \geq -1$ means $x + 1$ is positive or zero, so $x = -1 + \sqrt{y - 7}$
$g^{-1}(x) = -1 + \sqrt{x - 7}$
Answer: (a) $g(x) \geq 7$ (b) $g^{-1}(x) = -1 + \sqrt{x - 7}$

10 Where two tangents meet

The curve C has equation $y = x^2 - 3x - 6$.
The points A and B on C have x coordinates -3 and 0.
The tangents to C at A and at B meet at the point T. Find the coordinates of T.

$\frac{dy}{dx} = 2x - 3$. At A: gradient -9, A = $(-3, 12)$
Tangent at A: $y = -9x - 15$. At B: gradient -3, B = $(0, -6)$
Tangent at B: $y = -3x - 6$
Equate: $-9x - 15 = -3x - 6$, so $x = -\frac{3}{2}$ and $y = -\frac{3}{2}$
Answer: T $\left(-\frac{3}{2}, -\frac{3}{2}\right)$