Day 145 **IGCSE 10-a-day: Model Solutions**

1 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where $s = t^3 - 9t^2 + 24t$ $(t \geq 0)$

Find the distance between the two positions where P is at rest.

$v = \frac{ds}{dt} = 3t^2 - 18t + 24$

P is at rest when $v = 0$: $3t^2 - 18t + 24 = 0$

$3(t - 2)(t - 4) = 0$, so $t = 2$ or $t = 4$

$s(2) = 20$ and $s(4) = 16$

Distance $= |16 - 20| = 4$

Answer: 4 m

2 Terms of a quadratic sequence

The nth term of a sequence is $n^2 - n + 4$.

(a) Work out the 11th term.

(b) Is 94 a term of the sequence? You must show your working.

(a) $11^2 - 1 \times 11 + 4 = 114$

(b) Solve $n^2 - n + 4 = 94$, so $n^2 - n - 90 = 0$

Factorise or use the formula: $n = 10$ or $n = -9$

$n = 10$ is a positive whole number, so 94 is term number 10

Answer: (a) 114 (b) Yes, it is the 10th term

3 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 67° and angle CAQ = 70°.

(a) Work out the size of angle ABC.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 70° (alternate segment theorem)

(b) Angle ACB = angle PAB = 67° (alternate segment theorem)

Answer: (a) 70° (b) 67°

4 Increasing or decreasing at a point

$y = 2x^3 - 24x + 5$

At the point where $x = 1$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 - 24$

When $x = 1$: $\frac{dy}{dx} = -18$

-18 is negative, so the curve is decreasing

Answer: decreasing (gradient -18)

5 Radius from chord and height of arc

A circular arc has a chord AB of length 48 cm.

The greatest distance between the chord and the arc is 12 cm.

Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so $OM = r - 12$

$AM = 48 \div 2 = 24$. Pythagoras in triangle OMA: $r^2 = (r - 12)^2 + 24^2$

$r^2 = r^2 - 24r + 144 + 576$

$24r = 720$, so $r = 30$ cm

Answer: 30 cm

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6 Probability and quadratic equations

A bag contains x yellow discs and 4 blue discs.
Two discs are taken at random without replacement.

The probability that both are yellow is $\frac{3}{14}$.

(a) Show that $11x^2 - 35x - 36 = 0$.
(b) Hence work out the number of discs in the bag.

(a) P(both yellow) = $\frac{x}{x+4} \times \frac{x-1}{x+3} = \frac{3}{14}$

$14\,x(x-1) = 3(x+4)(x+3)$

$14x^2 - 14x = 3x^2 + 21x + 36$ so $11x^2 - 35x - 36 = 0$

(b) Solve: $x = 4$ or $x = -\frac{9}{11}$

x cannot be negative, so $x = 4$

Number of discs = 4 + 4 = 8

Answer: x = 4, so 8 discs in the bag

7 Find constants from a gradient

A curve has equation $y = ax^2 + \frac{b}{x}$, where a and b are constants.

The point (2, 10) lies on the curve and the gradient of the curve at this point is 13.
Find the values of a and b.

$\frac{dy}{dx} = 2ax - bx^{-2} = 2ax - \frac{b}{x^2}$

Point on curve: $4a + \frac{b}{2} = 10$. Gradient: $4a - \frac{b}{4} = 13$

Solving simultaneously gives $a = 3$ and $b = -4$

Answer: a = 3, b = −4

8 Conditional probability from a two-way table

The table shows how 117 students choose a sport.
A person from the group "Year 10" is picked at random. Work out the probability that this person chose Hockey.

Total in the group "Year 10" = 54

Number who chose Hockey = 14

Probability = $\frac{14}{54} = \frac{7}{27}$

Answer: $\frac{7}{27}$

9 Surds and fractions

Simplify $\frac{1}{3+\sqrt{3}} - \frac{1}{3-\sqrt{3}}$

Give your answer with a rational denominator.

Common denominator: $(3+\sqrt{3})(3-\sqrt{3}) = 9 - 3 = 6$

Numerator: $(3-\sqrt{3}) - (3+\sqrt{3})$

$= -2\sqrt{3}$

$\frac{-2\sqrt{3}}{6} = -\frac{\sqrt{3}}{3}$

Answer: $-\frac{\sqrt{3}}{3}$

10 Probability from a frequency tree

In a population of 20,000 people, 5% have a certain disease.
A test gives a positive result for 98% of people who have the disease and for 1% of people who do not.
A person is chosen at random and tests positive.
Work out the probability that they have the disease.

5% of 20,000 = 1,000 have the disease and 19,000 do not
Flagged and genuine: 1,000 × 98% = 980
Flagged but not: 19,000 × 1% = 190
Total flagged or positive = 980 + 190 = 1,170

Probability = 980 ÷ 1,170 = $\frac{98}{117}$

Answer: $\mathbf{\frac{98}{117}}$