Day 144 **IGCSE 10-a-day: Model Solutions**

1 Adding and scaling matrices

$A = \begin{pmatrix} 1 & -6 \\ 8 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} 8 & -2 \\ 5 & 2 \end{pmatrix}$.

Find the matrix X such that A + X = B.

$X = B - A$

$X = \begin{pmatrix} 8 & -2 \\ 5 & 2 \end{pmatrix} - \begin{pmatrix} 1 & -6 \\ 8 & -1 \end{pmatrix} = \begin{pmatrix} 7 & 4 \\ -3 & 3 \end{pmatrix}$

Answer: $\mathbf{X = \begin{pmatrix} 7 & 4 \\ -3 & 3 \end{pmatrix}}$

2 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 2.
45% of the juniors and 20% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 60 juniors and 40 seniors
45% of 60 = 27 and 20% of 40 = 8
Together: 35 out of 100
35 ÷ 100 × 100 = 35%
Answer: 35%

3 Expand then differentiate

$y = (4x - 2)^2$

Find $\frac{dy}{dx}$.

Expand: $y = 16x^2 - 16x + 4$
$16x^2 \rightarrow 32x$, $-16x \rightarrow -16$
$\frac{dy}{dx} = 32x - 16$

Answer: $\mathbf{\frac{dy}{dx} = 32x - 16}$

4 Stratified sampling

The table shows the number of students in each of four groups.
(a) A stratified sample of 30 is taken. Work out how many come from Year 7.
(b) A different stratified sample has 15 from Year 8. Work out the size of this sample.

Total = 200 + 125 + 225 + 200 = 750

(a) Year 7: $\frac{200}{750} \times 30 = 8$

(b) Fraction sampled = 15 ÷ 125 = $\frac{3}{25}$, so sample size = $\frac{3}{25} \times 750 = 90$

Answer: (a) 8 (b) 90

5 Laws of indices

Given that $\frac{8^4 \times 32^4}{2^2} = 2^n$,

find the value of n.

$8 = 2^3$, $32 = 2^5$
Numerator: $2^{12} \times 2^{20} = 2^{32}$
Denominator: 2^2
Subtract the powers: 32 − 2 = 30
Answer: n = 30

6 Reading a trigonometric graph

The diagram shows the graph of $y = a \cos bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 2, least value = 0
$c = \frac{1}{2}(2 + 0) = 1$ and $a = \frac{1}{2}(2 - 0) = 1$
One full cycle takes 360° (1 cycle between 0 and 360), so $b = 360 \div 360 = 1$
Answer: a = 1, b = 1, c = 1

7 Gradient function from a given point

A curve has equation $y = -2x^3 + 54x - 2$.
The point A $(-2, k)$ lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute $x = -2$ into the equation of the curve
$y = -94$ when $x = -2$, so $k = -94$
$\frac{dy}{dx} = -6x^2 + 54$
At A: gradient = 30
Answer: k = −94, gradient = 30

8 Composite functions

$f(x) = 4x + 3$ and $g(x) = 7x^2 - 3$
(a) Find fg(x).
(b) Solve fg(x) = 19

(a) $fg(x) = f(7x^2 - 3) = 4(7x^2 - 3) + 3$
$= 28x^2 - 9$
(b) $28x^2 - 9 = 19$, so $28x^2 = 28$
$x^2 = 1$, so $x = \pm 1$
Answer: (a) $28x^2 - 9$ (b) x = ±1

9 Number of solutions of f(x) = k

The diagram shows a sketch of the curve $y = -x^3 - 9x^2 - 24x - 1$ and the line $y = k$.
The line meets the curve at three different points.
Find the range of possible values of k.

$\frac{dy}{dx} = -3x^2 - 18x - 24 = 0$ gives $x = -4$ and $x = -2$
$y = 15$ at $x = -4$ and $y = 19$ at $x = -2$
The line crosses three times when it lies between the two turning-point values
$15 < k < 19$
Answer: 15 < k < 19

10 Loci and regions

ABCD is a rectangle with AB = 17 cm and AD = 13 cm.
A point P inside the rectangle is such that AP ≤ 12 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 12 cm: P is inside the quarter circle of radius 12 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area $= \frac{1}{2} \times \frac{1}{4} \times \pi \times 12^2 = 18\pi$ cm^2
Answer: 18π cm^2