Day 143 **IGCSE 10-a-day: Model Solutions**

1 Venn diagram: shaded region notation

Write down the set notation for the region shaded in the Venn diagram.

Identify which regions are shaded, using the circles X and Y
The shaded region is $Y' \cup X$
Answer: $Y' \cup X$

2 Fractions removed from a ratio

The ratio of blue to green sweets in a tin is 9 : 2.
Then $\frac{2}{3}$ of the blue and $\frac{1}{3}$ of the green are taken out.
Work out the ratio of blue to green now.

Left: $\frac{1}{3} \times 9 = 3$ of the blue part, $\frac{2}{3} \times 2 = \frac{4}{3}$ of the green part

$3 : \frac{4}{3}$
Multiply by 3: 9 : 4
Answer: 9 : 4

3 Expanding brackets with surds

Show that $(6 + 3\sqrt{3})(6 - 3\sqrt{3})$ is an integer.

Use the difference of two squares: $(6 + 3\sqrt{3})(6 - 3\sqrt{3}) = 36 - (3\sqrt{3})^2$
$= 36 - 27 = 9$
Answer: 9

4 Exponential growth and decay

A population of members of a website starts at 3,000 and grows by 25% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.25, double is 6,000
After 3 years: $3000 \times 1.25^3 = 5859.4$
After 4 years: $3000 \times 1.25^4 = 7324.2$
Answer: 4 years

5 Stationary points of a cubic

A curve has equation $y = 2x^3 - 15x^2 + 24x + 5$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = 6x^2 - 30x + 24$

Stationary points: $\frac{dy}{dx} = 0$, so $6(x - 1)(x - 4) = 0$

$x = 1$ or $x = 4$
$y = 16$ when $x = 1$ and $y = -11$ when $x = 4$
Answer: (1, 16) and (4, −11)

6 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out angle ACB, marked θ, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $\cos\theta = \frac{5^2 + 5^2 - 8^2}{2 \times 5 \times 5} = \frac{-14}{50}$
$\theta = 106.3°$ (1 d.p.)

(b) Area $= \frac{1}{2} \times 5 \times 5 \times \sin 106.3°$

$= 12.0\ \text{cm}^2$ (3 s.f.)
Answer: (a) 106.3° (b) 12.0 cm^2

7 Bounds of a compound expression

p = 7.3 and r = 6.6, each correct to 1 decimal place.
q = 4.64, correct to 2 decimal places.

Work out the upper bound of $\frac{p+q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom
Top: UB p + UB q = 7.35 + 4.645 = 11.995
Bottom: LB r = 6.55
11.995 ÷ 6.55 = 1.83
Answer: 1.83

8 Curves that touch

The curves $y = x^2 - 2x - 4$ and $y = 2x^2 + 4x + 5$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $x^2 + 6x + 9 = 0$
That is $(x + 3)^2 = 0$ with a repeated root $x = -3$
$y = 11$, so the point is $(-3, 11)$
$\frac{dy}{dx} = 2x - 2$ on the first curve: gradient -8
Tangent: $y = -8x - 13$
Answer: $(-3, 11)$, $y = -8x - 13$

9 Algebraic proof (harder)

Show algebraically that the sum of the cubes of any three consecutive integers is a multiple of 9.

$(m - 1)^3 + m^3 + (m + 1)^3 = 3m^3 + 6m = 3m(m^2 + 2)$
If m is a multiple of 3, then $3m(m^2 + 2)$ is a multiple of 9
If $m = 3m + 1$ or $m = 3m - 1$, then $m^2 + 2 = 9m^2 \pm 6m + 3 = 3(3m^2 \pm 2m + 1)$, a multiple of 3
In every case $3m(m^2 + 2)$ contains two factors of 3, so it is a multiple of 9
Answer: Proved

10 Three stationary points of a quartic

$y = -x^4 + 2x^2 - 2$
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = -4x^3 + 4x = -4x(x - 1)(x + 1)$
$\frac{dy}{dx} = 0$ gives $x = -1$, $x = 0$ or $x = 1$
Points: $(-1, -1)$, $(0, -2)$, $(1, -1)$
$\frac{d^2y}{dx^2} = -12x^2 + 4$
$x = -1$: $\frac{d^2y}{dx^2} = -8$; $x = 0$: $\frac{d^2y}{dx^2} = 4$; $x = 1$: $\frac{d^2y}{dx^2} = -8$
So $(-1, -1)$ is a maximum, $(0, -2)$ is a minimum, $(1, -1)$ is a maximum
Answer: $(-1, -1)$ maximum; $(0, -2)$ minimum; $(1, -1)$ maximum