Day 142 **IGCSE 10-a-day: Model Solutions**

1 Error intervals

The number d is 440 correct to the nearest 10.
Write down the error interval for d.

The rounding unit is 10, so the half-unit is 5
Lower bound = 440 − 5 = 435; upper bound = 440 + 5 = 445
Answer: $435 \leq d < 445$

2 Direct proportion formulae

C is directly proportional to the square root of d.
When d = 36, C = 18.
(a) Find a formula for C in terms of d.
(b) Work out the positive value of d when C = 15.

(a) $C = k\sqrt{d}$
$18 = k \times 6$, so $k = 18 \div 6 = 3$
$C = 3\sqrt{d}$
(b) $15 = 3\sqrt{d}$, so $\sqrt{d} = 15 \div 3 = 5$, giving $d = 25$
Answer: (a) $C = 3\sqrt{d}$ (b) $d = 25$

3 Fastest rate of growth

The number of visitors to a website, N, after t days is modelled by
$N = -2t^3 + 18t^2 + 5t + 100$ for $0 \leq t \leq 12$
Find the time at which N is increasing most rapidly, and the rate of increase at this time.

$\frac{dN}{dt} = -6t^2 + 36t + 5$

Rate is greatest when $\frac{d^2N}{dt^2} = -12t + 36 = 0$, so $t = 3$

$\frac{d^2N}{dt^2}$ is decreasing (gradient −12), so the rate is a maximum

$\frac{dN}{dt} = 59$ when $t = 3$

Answer: t = 3, rate 59 per day

4 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} 0 \\ -6 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} -6 \\ -5 \end{pmatrix}$.
Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix} 0 \\ -12 \end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix} 5 \\ -3 \end{pmatrix} + \begin{pmatrix} 0 \\ -12 \end{pmatrix} - \begin{pmatrix} -6 \\ -5 \end{pmatrix} = \begin{pmatrix} 11 \\ -10 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{11} \\ \mathbf{-10} \end{pmatrix}$

5 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = (2x)°, angle ACB = (x + 27)° and angle CAQ = 52°.
(a) Work out the value of x.
(b) Work out the size of angle BAC.
The diagram is not drawn accurately.

(a) Angle ACB = angle PAB (alternate segment theorem)
x + 27 = 2x
x = 27
(b) Angle PAB = 54°
Angles on the straight line PAQ: angle BAC = 180 − 54 − 52 = 74°
Answer: (a) x = 27 (b) 74°

6 Gradient function from a given point

A curve has equation $y = -2x^3 + 12x^2 - 18x$.
The point A (−1, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute x = −1 into the equation of the curve
y = 32 when x = −1, so k = 32
$\frac{dy}{dx} = -6x^2 + 24x - 18$
At A: gradient = −48
Answer: k = 32, gradient = −48

7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and a > 0.
The curve passes through the points (1, 16) and (3, 64).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 16$ and $k\,a^3 = 64$
Divide: $a^2 = \frac{64}{16} = 4$
a = 2 (positive, since a > 0)
From the first point: k × 2 = 16, so k = 8
Answer: k = 8, a = 2

8 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = t^3 - 6t^2 + 21t + 5$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 3t^2 - 12t + 21 = 3(t - 2)^2 + 9$
A square is never negative, so v ≥ 9 > 0 and P is never at rest
Least value of v occurs when t = 2, giving 9
Answer: (b) 9 m/s when t = 2 s

9 Recurring decimals

Without using a calculator, work out $0.\dot{1}\dot{6} + 0.\dot{6}\dot{0}$
Give your answer as a fraction in its simplest form.

$0.\dot{1}\dot{6} = \frac{16}{99}$
$0.\dot{6}\dot{0} = \frac{20}{33}$
$\frac{16}{99} + \frac{20}{33} = \frac{76}{99}$
Answer: $\frac{76}{99}$

10 Estimated mean with an unknown frequency

The table gives information about the masses of some eggs.
An estimate for the mean is 20.5.
Work out the value of x.

Mid-points: 12.5, 17.5, 22.5, 27.5, 32.5
Estimated mean = $\Sigma fm \div \Sigma f$, so $(662.5 + 22.5x) \div (33 + x) = 20.5$
$662.5 + 22.5x = 20.5(33 + x) = 676.5 + 20.5x$
$(22.5 - 20.5)x = 676.5 - 662.5$
$2x = 14$, so $x = 7$
Answer: x = 7