Day 141 **IGCSE 10-a-day: Model Solutions**

1 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -2t^2 + 4t + 5$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 5. State whether it is moving up or down.

$\frac{dh}{dt} = -4t + 4 = 0$ gives $t = 1$

$\frac{d^2h}{dt^2} = -4 < 0$ so maximum. $h = 7$

When $t = 5$: $\frac{dh}{dt} = 4 - 4 \times 5 = -16$

Answer: (a) 7 m (b) −16 m/s, moving down

2 Writing inequalities from a situation

On a minibus there are x adults and y children.

There are at most 60 people. There are at least 3 children for every adult. There are at least 4 adults.

(a) Write down three inequalities for x and y.

(b) Find the greatest number of children on the minibus.

(a) At most 60 people: $x + y \leq 60$

At least 3 children for every adult: $y \geq 3x$

At least 4 adults: $x \geq 4$

(b) The most children occurs with the fewest adults: $x = 4$

$y \leq 60 - 4 = 56$ and $y \geq 12$, so $y = 56$ is allowed

Answer: (a) $x + y \leq 60$, $y \geq 3x$, $x \geq 4$ (b) 56

3 Factorising fully

Factorise fully $5x^2 + 10x - 75$

Take out the common factor 5: $5(x^2 + 2x - 15)$

Factorise the quadratic: $5(x - 3)(x + 5)$

Answer: $5(x - 3)(x + 5)$

4 Tree diagrams without replacement

A bag contains 4 orange cubes, 3 purple cubes and 3 pink cubes.

Two cubes are taken at random without replacement.

Work out the probability that (a) both are the same colour, (b) at least one is purple.

(a) Add the ways: $\frac{4}{10} \times \frac{3}{9} + \frac{3}{10} \times \frac{2}{9} + \frac{3}{10} \times \frac{2}{9} = \frac{4}{15}$

(b) P(at least one purple) = 1 − P(none) = $1 - \frac{7}{10} \times \frac{6}{9} = \frac{8}{15}$

Answer: (a) $\frac{4}{15}$ (b) $\frac{8}{15}$

5 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by

$v = 20t - 5t^2$ for $0 \leq t \leq 4$

Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -10t + 20$

Greatest speed when $a = 0$: $-10t + 20 = 0$, so $t = 2$

$\frac{d^2v}{dt^2} = -10 < 0$, so this is a maximum

$v = 20 \times 2 - 5 \times 2^2 = 20$

Answer: t = 2 s, greatest speed 20 m/s

6 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 5 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 10 ÷ 5 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 5 \times 10 + 10 \times 12 + \frac{1}{2} \times 5 \times 10 = 25 + 120 + 25$
= 170
Answer: (a) 2 m/s^2 (b) 170 m

7 Maximum revenue

A company sells x thousand items each month. The price of each item is £56 − 4x.
The monthly revenue is £R thousand.
(a) Show that R = 56x − 4x^2.
(b) Find the maximum monthly revenue, showing that it is a maximum.

R = x × (56 − 4x) = 56x − 4x^2
$\frac{dR}{dx} = -8x + 56 = 0$ gives x = 7
$\frac{d^2R}{dx^2} = -8 < 0$, so a maximum
R = 56 × 7 − 4 × 7^2 = 196
Answer: £196 thousand

8 nth term of a cubic sequence

Here are the first five terms of a sequence.
4, 12, 32, 70, 132
The nth term is of the form an^3 + bn + c.
Find the values of a, b and c.

1st differences: 8, 20, 38, 62. 2nd: 12, 18, 24. 3rd: 6, 6
Third differences = 6a = 6 so a = 1
Subtract n^3: 3, 4, 5, 6, 7
This is linear with common difference 1, so b = 1, and c = 3 − 1 = 2
Answer: a = 1, b = 1, c = 2

9 Fraction problems

A fuel tank is $\frac{2}{5}$ full.

72 more litres of petrol are added and the fuel tank is then $\frac{5}{8}$ full.

Work out the capacity of the fuel tank.

Fraction added = $\frac{5}{8} - \frac{2}{5} = \frac{9}{40}$

$\frac{9}{40}$ of the capacity is 72 litres

Capacity = 72 ÷ $\frac{9}{40}$ = 320
Answer: 320 litres

10 Vectors: intersecting lines

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on OB with ON : NB = 2 : 3. AN and BM intersect at X.
(a) Find BM in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Find OX in terms of $\mathbf{a}$ and $\mathbf{b}$.
The diagram is not drawn accurately.

(a) OM = $\frac{1}{5}\mathbf{a}$, so BM = $-\mathbf{b} + \frac{1}{5}\mathbf{a} = \frac{1}{5}\mathbf{a} - \mathbf{b}$

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{5}s\mathbf{a} + (1 - s)\mathbf{b}$

X is on AN: OX = OA + t AN = OA + t(ON − OA) = $(1 - t)\mathbf{a} + \frac{2}{5}t\mathbf{b}$

Equate: $\frac{1}{5}s = 1 - t$ and $1 - s = \frac{2}{5}t$

Solving: $s = \frac{15}{23}$ and $t = \frac{20}{23}$

OX = $\frac{3}{23}\mathbf{a} + \frac{8}{23}\mathbf{b}$

Answer: (a) $\frac{1}{5}\mathbf{a} - \mathbf{b}$ (b) OX = $\frac{3}{23}\mathbf{a} + \frac{8}{23}\mathbf{b}$