Day 140 **IGCSE 10-a-day: Model Solutions**

1 Describe a graph transformation

Describe fully the single transformation that maps the graph of y = f(x) onto the graph of
(a) y = f(x) + 2
(b) y = f(2x)

(a) Adding 2 to f(x) moves the graph up 2 units
(b) Every x value is divided by 2

Answer: (a) A translation by the vector $\begin{pmatrix}0\\2\end{pmatrix}$ (b) A stretch parallel to the x-axis with scale factor $\frac{1}{2}$

2 Divide then differentiate

$y = \frac{2x^4 - 7x^3 - 5x^2}{x^3}$

Find $\frac{dy}{dx}$.

Divide each term by x^3: $y = 2x - 7 - 5x^{-1}$
$2x \to 2$, $-5x^{-1} \to 5x^{-2}$
$\frac{dy}{dx} = 2 + 5x^{-2}$

Answer: $\frac{dy}{dx} = 2 + \frac{5}{x^2}$

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 49°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 49°
Angle COB is an exterior angle of triangle OAC: 49 + 49 = 98°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 98°

4 Expanding brackets with surds

Expand and simplify $(3 - 3\sqrt{10})^2$
Give your answer in the form $p + q\sqrt{10}$

$(3 - 3\sqrt{10})^2 = 9 - 18\sqrt{10} + 90$
$= 99 - 18\sqrt{10}$
Answer: $99 - 18\sqrt{10}$

5 Tangent to a circle

A circle has equation $(x - 1)^2 + (y + 3)^2 = 65$.
The point P(2, −11) lies on the circle.
Find an equation of the tangent to the circle at P.
Give your answer in the form ax + by = c, where a, b and c are integers.

The centre is (1, −3). The gradient of the radius to P is $\frac{-11 - (-3)}{2 - 1} = -8$

The tangent is perpendicular to the radius, so its gradient is $\frac{1}{8}$. Using P: $y + 11 = \frac{1}{8}(x - 2)$

Multiply by 8: $8(y + 11) = 1(x - 2)$
Expand and rearrange: $-x + 8y = -90$
Multiply by −1: $x - 8y = 90$
Answer: $x - 8y = 90$

6 Sample space and conditional probability

A fair 6-sided dice and a spinner numbered 1 to 5 are each thrown once. The first score and the second score are recorded.

(a) Work out the probability that the product is even.

(b) Given that at least one number is 3, work out the probability that the product is even.

(a) There are 6 × 5 = 30 equally likely outcomes, and 21 of them satisfy: the product is even

Probability = $\frac{7}{10}$

(b) 10 outcomes satisfy: at least one number is 3. Of these, 5 also satisfy: the product is even

Probability = $\frac{1}{2}$

Answer: (a) $\frac{7}{10}$ (b) $\frac{1}{2}$

7 Triangle shapes from position vectors

A = (2, 2), B = (5, 4) and C = (0, 5).

Show that triangle ABC is isosceles and right-angled.

$\mathbf{AB} = \begin{pmatrix} 3 \\ 2 \end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix} -2 \\ 3 \end{pmatrix}$

$|AB|^2 = 9 + 4 = 13$ and $|AC|^2 = 4 + 9 = 13$

$|AB| = |AC| = \sqrt{13}$, so the triangle is isosceles

$\mathbf{BC} = \begin{pmatrix} -5 \\ 1 \end{pmatrix}$ so $|BC|^2 = 25 + 1 = 26$

$|AB|^2 + |AC|^2 = 13 + 13 = 26 = |BC|^2$, so the angle at A is 90° (Pythagoras)

Answer: $|\mathbf{AB}| = |\mathbf{AC}|$ and $\mathbf{AB}^2 + \mathbf{AC}^2 = \mathbf{BC}^2$

8 Fencing with a partition

A rectangular enclosure is x metres wide and y metres long. It is fenced all round and divided into three equal parts by two fences each of length x metres, parallel to the width.

The total length of fencing used is 48 m.

(a) Show that the area of the enclosure is $A = -2x^2 + 24x$.

(b) Find the maximum possible area, justifying that it is a maximum.

Fencing: $4x + 2y = 48$, so $y = \frac{48 - 4x}{2}$

$A = xy = \frac{x(48 - 4x)}{2} = -2x^2 + 24x$

$\frac{dA}{dx} = -4x + 24 = 0$ gives $x = 6$

$\frac{d^2A}{dx^2} = -4 < 0$, so a maximum

$A = 72$

Answer: 72 m²

9 Percentages with an unknown rate

The price of a share is increased by x%.
The new price is then decreased by x%.
The final price is 16% less than the original price.
Work out the value of x.

Let the original price be 1. Multipliers: $(1 + \frac{x}{100})$ and $(1 - \frac{x}{100})$

$(1 + \frac{x}{100})(1 - \frac{x}{100}) = 1 - \frac{x^2}{10000}$

Final price is 16% less, so the multiplier is 0.84: $1 - \frac{x^2}{10000} = 0.84$

$\frac{x^2}{10000} = 0.16$, so $x^2 = 1600$ and $x = 40$

Answer: x = 40

10 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - 3x^2 - 4x + 3$ at the point P where $x = 2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 6x - 4$, so at P the gradient is -4 and $y = -9$

Tangent: $y = -4x - 1$

Intersection: $x^3 - 3x^2 + 4 = 0$

$x = 2$ is a repeated root, so the roots are 2, 2 and x_Q

Sum of roots = 3, so $4 + x_Q = 3$ and $x_Q = -1$

$y = 3$

Answer: (−1, 3)