Day 14 **IGCSE 10-a-day: Model Solutions**

1 Tangent at a turning point

The graph of $y = 3x^2 + 18x - 3$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = 6x + 18$

At P, $\frac{dy}{dx} = 0$, so $6x + 18 = 0$ and $x = -3$

$y = -30$ at P

The gradient is 0, so the tangent is horizontal: $y = -30$

Answer: $y = -30$

2 Two picks without replacement

A bag contains 3 green, 7 blue and 2 red beads.
Two beads are taken at random without replacement.
Work out the probability that at least one is green.

P(at least one green) = 1 − P(neither is green)

P(neither) = $\frac{9}{12} \times \frac{8}{11}$

$= 1 - \frac{6}{11} = \frac{5}{11}$

Answer: $\frac{5}{11}$

3 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 43°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 43°
Angle COB is an exterior angle of triangle OAC: 43 + 43 = 86°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 86°

4 Weighted percentages

The ratio of juniors to seniors in a school is 5 : 4.
30% of the juniors and 75% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 180 students in total: 100 juniors and 80 seniors
30% of 100 = 30 and 75% of 80 = 60
Together: 90 out of 180
90 ÷ 180 × 100 = 50%
Answer: 50%

5 Stationary points of y = ax + b/x

$y = 4x + \frac{64}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 4 - \frac{64}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{64}{x^2} = 4$, so $x^2 = 16$ and $x = 4$ or $x = -4$

$y = 32$ when $x = 4$ and $y = -32$ when $x = -4$

$\frac{d^2y}{dx^2} = \frac{128}{x^3}$

$x = 4$: $\frac{d^2y}{dx^2} = 2$ so minimum; $x = -4$: $\frac{d^2y}{dx^2} = -2$ so maximum

Answer: (4, 32) minimum; (−4, −32) maximum

6 Rationalising the denominator

Write $\frac{14}{2 + \sqrt{3}}$ in the form $p + q\sqrt{3}$

where p and q are integers.

Multiply top and bottom by $(2 - \sqrt{3})$:

$\frac{14}{2 + \sqrt{3}} = \frac{14(2 - \sqrt{3})}{(2 + \sqrt{3})(2 - \sqrt{3})}$

Denominator: $4 - 3 = 1$

$= 28 - 14\sqrt{3}$

Answer: $28 - 14\sqrt{3}$

7 Bounds: finding an unknown length

A rectangle has area 57 cm^2, correct to 2 significant figures.
Its length is 6.900000000000000355271367880050092935562133789062 5 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length

Area: $56.5 \leq A < 57.5$. Length: $6.850000000000000355271367880050092935562133789062 5 \leq l < 6.950000000000000355271367880050092935562133789062 5$

UB width = UB area ÷ LB length = 57.5 ÷ 6.850000000000000355271367880050092935562133789062 5 = 8.39

Answer: 8.39 cm

8 Stationary points of a cubic

A curve has equation $y = x^3 - 6x^2 + 9x + 3$.
Find the coordinates of the stationary points of the curve.

$\frac{dy}{dx} = 3x^2 - 12x + 9$

Stationary points: $\frac{dy}{dx} = 0$, so $3(x - 1)(x - 3) = 0$

$x = 1$ or $x = 3$

$y = 7$ when $x = 1$ and $y = 3$ when $x = 3$

Answer: (1, 7) and (3, 3)

9 Transformations of graphs

The point P(12, −2) lies on the graph of y = f(x).
Write down the coordinates of the image of P on the graph of

(a) y = f(2x) (b) y = $\frac{1}{2}$f(x)

(a) y = f(2x) is a horizontal stretch with scale factor $\frac{1}{2}$, so the x coordinate is halved: (6, −2)

(b) y = $\frac{1}{2}$f(x) is a vertical stretch with scale factor $\frac{1}{2}$, so the y coordinate is halved: (12, −1)

Answer: (a) (6, −2) (b) (12, −1)

10 Frustum of a cone

A cone is cut parallel to its base to make a smaller cone and a frustum.
The frustum has height 3 cm, top radius 4 cm and base radius 8 cm.
Work out the volume of the frustum. Give your answer in terms of π.

Let the height of the small cone removed be h cm. The cones are similar, so $\frac{h}{h+3} = \frac{4}{8}$

8 h = 4(h + 3), so 4 h = 12 and h = 3

Large cone: $\frac{1}{3} \times \pi \times 8^2 \times 6 = 128\pi$

Small cone: $\frac{1}{3} \times \pi \times 4^2 \times 3 = 16\pi$

Frustum = $128\pi - 16\pi = 112\pi$ cm^3

Answer: 112π cm^3