Day 139 **IGCSE 10-a-day: Model Solutions**

1 Two numbers: maximise or minimise

The positive numbers x and y satisfy $2x + y = 28$.
Find the greatest possible value of xy, showing that it is the greatest.

$y = 28 - 2x$, so $xy = x(28 - 2x) = -2x^2 + 28x$
$\frac{d}{dx}(xy) = -4x + 28 = 0$ gives $x = 7$
Second derivative $= -4 < 0$, so a maximum
$y = 14$, so the greatest value of xy is $7 \times 14 = 98$
Answer: 98

2 Laws of indices with algebra

Simplify fully $\frac{6x^6y^3 \times 7x^2y^4}{6x^3y^2}$

Give your answer in the form kx^ay^b

Numbers: $6 \times 7 \div 6 = 7$
Powers of x: $6 + 2 - 3 = 5$
Powers of y: $3 + 4 - 2 = 5$
$= 7x^5y^5$
Answer: $7x^5y^5$

3 Sets of numbers: lists and counting

ξ = integers from 1 to 24
A = the numbers greater than 20
B = the prime numbers
(a) List the elements of $A \cap B'$.
(b) Find $n(A' \cap B')$.

A = 21, 22, 23, 24
B = 2, 3, 5, 7, 11, 13, 17, 19, 23
(a) $A \cap B' = 21, 22, 24$
(b) $A \cup B = 2, 3, 5, 7, 11, 13, 17, 19, 21, 22, 23, 24$ which has 12 elements, so $n(A' \cap B') = 24 - 12 = 12$
Answer: (a) 21, 22, 24 (b) 12

4 Direct proportion formulae

E is directly proportional to the cube of v.
When v = 3, E = 108.
(a) Find a formula for E in terms of v.
(b) Work out the positive value of v when E = 32.

(a) $E = kv^3$
$108 = k \times 27$, so $k = 108 \div 27 = 4$
$E = 4v^3$
(b) $32 = 4v^3$, so $v^3 = 32 \div 4 = 8$, giving $v = 2$
Answer: (a) $E = 4v^3$ (b) $v = 2$

5 Using a quadratic nth term

The nth term of a sequence is $2n^2 - 4n + 8$.
(a) Work out the 9th term.
(b) One term of the sequence is 78. Work out the value of n for this term.

(a) $2 \times 9^2 - 4 \times 9 + 8 = 134$
(b) $2n^2 - 4n + 8 = 78$ so $2n^2 - 4n - 70 = 0$
Solve (factorise or use the formula): $n = 7$ or $n = -5$. n must be a positive integer, so $n = 7$
Answer: (a) 134 (b) n = 7

6 Rationalising the denominator

Rationalise the denominator of $\frac{3}{3\sqrt{2}}$

Give your answer in its simplest form.

Multiply top and bottom by $\sqrt{2}$:

$\frac{3}{3\sqrt{2}} = \frac{3\sqrt{2}}{6}$

$= \frac{\sqrt{2}}{2}$

Answer: $\frac{\sqrt{2}}{2}$

7 Minimum running cost

The cost, C pence per kilometre, of running a van at a steady speed of v km/h is modelled by

$C = \frac{1}{10}v + \frac{810}{v}$ $(v > 0)$

Find the speed at which the cost per kilometre is least, and show that it is a minimum.

Work out the least cost per kilometre.

$\frac{dC}{dv} = \frac{1}{10} - 810v^{-2} = 0$, so $v^2 = 8100$ and $v = 90$

$\frac{d^2C}{dv^2} = 1620v^{-3} = \frac{1}{450} > 0$, so a minimum

$C = \frac{810}{90} + \frac{90}{10} = 18$

Answer: v = 90 km/h, C = 18 pence per km

8 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)

(a) Work out the length of the diagonal AG.

(b) Work out the angle between AG and the base ABCD, to 1 decimal place.

The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 8^2 + 1^2 = 65$

$AG^2 = AC^2 + CG^2 = 65 + 4^2 = 81$

$AG = 9$ cm

(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{65}}$

angle GAC = 26.4° (1 d.p.)

Answer: (a) 9 cm (b) 26.4°

9 Rectangle under a parabola

The diagram shows the curve $y = 48 - x^2$. Rectangle ABCD has A, B on the x-axis and C, D on the curve. B is (x, 0).

(a) Show that the area of ABCD is $-2x^3 + 96x$.

(b) Find the maximum area of ABCD, showing that it is a maximum.

AB = 2x and BC = $48 - x^2$, so area = $2x(48 - x^2) = -2x^3 + 96x$

$\frac{dA}{dx} = -6x^2 + 96 = 0$, so $x^2 = 16$ and $x = 4$ $(x > 0)$

$\frac{d^2A}{dx^2} = -12x < 0$ when $x = 4$, so a maximum

Area = $96 \times 4 - 2 \times 4^3 = 256$

Answer: 256

10 Combined transformations

The point P(3, 6) is rotated 180° about the origin.
The image is then reflected in the line $y = -x$.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = $(-3, -6)$
After the second transformation: P'' = (6, 3)
(b) Follow a general point (x, y) through both transformations: $(x, y) \rightarrow (-x, -y) \rightarrow (y, x)$
The single mapping is $(x, y) \rightarrow (y, x)$
This is a reflection in the line $y = x$
Answer: (a) (6, 3) (b) Reflection in the line $y = x$