Day 138 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(a - b)^2 - (a + b)^2$
when $a = -3$, $b = -3$ and $c = 2$.

Substitute the values: $(-3 - (-3))^2 - (-3 + (-3))^2$
$= 0 - 36$
$= -36$
Answer: -36

2 Find unknown constants in a function

$f(x) = ax + b$, where a and b are constants.
$f(0) = -9$ and $f(1) = -6$.
Find the value of a and the value of b.

$0 + b = -9$ and $a + b = -6$
Subtract: $-a = -3$, so $a = 3$
$b = -9 - 3 \times 0 = -9$
Answer: $a = 3$, $b = -9$

3 Weighted percentages

The ratio of juniors to seniors in a school is 3 : 2.
45% of the juniors and 40% of the seniors play a sport.
What percentage of all of these students play a sport?

Imagine 100 students in total: 60 juniors and 40 seniors
45% of 60 = 27 and 40% of 40 = 16
Together: 43 out of 100
$43 \div 100 \times 100 = 43\%$
Answer: 43%

4 Simultaneous equations

Solve the simultaneous equations
$4x + 2y = -14$
$x + 6y = 24$

To match the x terms: (2) × 4 gives $4x + 24y = 96$
Subtract the equations to eliminate x: $-22y = -110$
$y = 5$
Substitute $y = 5$ into (1): $4x + 10 = -14$
$4x = -24$, so $x = -6$
Answer: $x = -6$, $y = 5$

5 Square roots as powers

$y = -6x^{5/2} + 2x - \frac{2}{\sqrt{x}}$

Find $\frac{dy}{dx}$. Give each term of your answer in the form ax^n.

Write as powers of x: $y = -6x^{5/2} + 2x - 2x^{-1/2}$
$-6x^{5/2} \rightarrow -15x^{3/2}$, $2x \rightarrow 2$, $-2x^{-1/2} \rightarrow x^{-3/2}$
$\frac{dy}{dx} = -15x^{3/2} + 2 + x^{-3/2}$
Answer: $\frac{dy}{dx} = -15x^{3/2} + 2 + x^{-3/2}$

6 Reading a trigonometric graph

The diagram shows the graph of $y = a \sin bx° + c$ for $0 \leq x \leq 360$.
a, b and c are integers.
Find the values of a, b and c.

Greatest value = 6, least value = -2
$c = \frac{1}{2}(6 + (-2)) = 2$ and $a = \frac{1}{2}(6 - (-2)) = 4$
One full cycle takes 360° (1 cycle between 0 and 360), so $b = 360 \div 360 = 1$
Answer: a = 4, b = 1, c = 2

7 Stationary points of a quartic

A curve has equation $y = x^4 - 8x^2 - 4$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 16x = 4x(x - 2)(x + 2)$
$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 2$ or $x = -2$
$y = -4$ when $x = 0$, and $y = -20$ when $x = \pm 2$
$\frac{d^2y}{dx^2} = 12x^2 - 16$: it is -16 at $x = 0$ (maximum) and 32 at $x = \pm 2$ (minimum)
Answer: (0, −4) maximum; (2, −20) and (−2, −20) minima

8 Growth and decay

Lena puts £4,000 into a savings account.
The account pays 12% compound interest each year.
After how many whole years will there first be more than £6,000 in the account? (Calculator)

Multiplier each year = 1.12
After 3 years: £4,000 × 1.12^3 = £5,619.71
After 4 years: £4,000 × 1.12^4 = £6,294.08
So the first time it is more than £6,000 is after 4 years
Answer: 4 years

9 Increasing for all x

$f(x) = x^3 + kx^2 + 75x + 3$, where k is a constant.
Given that f is an increasing function for all values of x, find the range of possible values of k.

$f'(x) = 3x^2 + 2kx + 75$
f is increasing for all x when $f'(x) > 0$ for all x, so the quadratic has no real roots
Discriminant: $(2k)^2 - 4 \times 3 \times 75 < 0$, so $4k^2 < 900$
$k^2 < 225$, so $-15 < k < 15$
Answer: $-15 < k < 15$

10 Loci and regions

ABCD is a rectangle with AB = 12 cm and AD = 12 cm.
A point P inside the rectangle is such that AP ≤ 10 cm and P is nearer to AB than to AD.
Work out the area of the region in which P can lie. Give your answer in terms of π.

AP ≤ 10 cm: P is inside the quarter circle of radius 10 cm centred at A (it fits inside the rectangle)
Nearer to AB than AD: P is on the AB side of the angle bisector of angle DAB, which is the line through A at 45° to AB
The bisector cuts the quarter circle into two equal halves
Area = $\frac{1}{2} \times \frac{1}{4} \times \pi \times 10^2 = \frac{25}{2}\pi$ cm^2
Answer: $\frac{25}{2}\pi$ cm^2