**Day 137** **IGCSE 10-a-day: Model Solutions**

## 1 Velocity and acceleration

A particle P moves in a straight line. After t seconds its displacement from a fixed point O is s metres, where

$s = 3t^2 - 5t + 5 + \frac{8}{t}$ $(t > 0)$

Find (a) the velocity of P, (b) the acceleration of P, when t = 1.

$v = \frac{ds}{dt} = 6t - 5 - 8t^{-2}$

$a = \frac{dv}{dt} = \frac{d^2s}{dt^2} = 6 + 16t^{-3}$

When t = 1: v = −7 and a = 22

**Answer: (a) −7 m/s (b) 22 m/s$^2$**

## 2 Inequalities from a region

Region R is shaded. Solid boundaries are included, dashed ones are not.

(a) Write down the three inequalities that define R.

(b) How many points with integer coordinates lie in R?

The vertical line is x = 2, the horizontal line is y = 2

The sloping line passes through (6, 2) and (2, 7): 5x + 4y = 38

R is to the right of x = 2, above y = 2 and below the sloping line. Solid lines are included, dashed lines are not: $x > 2$, $y \geq 2$, $5x + 4y < 38$

Count the integer points column by column: 4 + 3 + 2 = 9

**Answer: (a) $x > 2$, $y \geq 2$, $5x + 4y < 38$ (b) 9**

## 3 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)

(a) Work out angle ACB, marked θ, to 1 decimal place.

(b) Work out the area of triangle ABC to 3 significant figures.

The diagram is not drawn accurately.

(a) $\cos\theta = \frac{10^2 + 11^2 - 14^2}{2 \times 10 \times 11} = \frac{25}{220}$

θ = 83.5° (1 d.p.)

(b) Area $= \frac{1}{2} \times 10 \times 11 \times \sin 83.5°$

= 54.6 cm$^2$ (3 s.f.)

**Answer: (a) 83.5° (b) 54.6 cm$^2$**

## 4 Simultaneous equations

Solve the simultaneous equations

4x + 2y = 10

−4x + 6y = −2

To match the y terms: (1) × 3 gives 12x + 6y = 30

Subtract the equations to eliminate y: 16x = 32

x = 2

Substitute x = 2 into (1): 2y + 8 = 10

2y = 2, so y = 1

**Answer: x = 2, y = 1**

## 5 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by

$v = 36t - 3t^2$ for $0 \leq t \leq 12$

Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -6t + 36$

Greatest speed when $a = 0$: $-6t + 36 = 0$, so $t = 6$

$\frac{d^2v}{dt^2} = -6 < 0$, so this is a maximum

$v = 36 \times 6 - 3 \times 6^2 = 108$

**Answer: t = 6 s, greatest speed 108 m/s**

## 6 Enlargement

Triangle T has vertices A(3, 1), B(−5, −3), C(5, 7).

T is enlarged with scale factor $\frac{1}{2}$ and centre (1, 1).

(a) Write down the coordinates of the image of A.

(b) The area of T is 18 square units. Work out the area of the image of T.

(a) Vector from the centre to A = (3 − 1, 1 − 1) = (2, 0)

Multiply by $\frac{1}{2}$: (1, 0)

Image of A = centre + this vector = (2, 1)

(b) Area scale factor = $(\frac{1}{2})^2 = \frac{1}{4}$

Area = $18 \times \frac{1}{4} = \frac{9}{2}$ square units

**Answer: (a) (2, 1) (b) $\frac{9}{2}$**

## 7 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where

$s = 2t^3 - 6t^2 + 11t - 2$

(a) Show that the particle is never at rest.

(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 12t + 11 = 6(t - 1)^2 + 5$

A square is never negative, so $v \geq 5 > 0$ and P is never at rest

Least value of v occurs when t = 1, giving 5

**Answer: (b) 5 m/s when t = 1 s**

## 8 Inverse proportion

V is inversely proportional to r.

When r = 4, V = 3.

Find the positive value of r when V = 6.

$V = \frac{k}{r}$

$3 = \frac{k}{4}$, so $k = 3 \times 4 = 12$

$6 = \frac{12}{r}$, so $r = \frac{12}{6} = 2$

**Answer: r = 2**

## 9 Bounds of a compound expression

p = 6.3 and r = 7.1, each correct to 1 decimal place.
q = 3.86, correct to 2 decimal places.

Work out the upper bound of $\frac{p - q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the upper bound use UB of the top and LB of the bottom
Top: UB p − LB q = 6.35 − 3.855 = 2.495
Bottom: LB r = 7.05
2.495 ÷ 7.05 = 0.354
**Answer: 0.354**

## 10 Index equations

Solve $4^{2x + 2} = 2^{2x + 4}$

Write both sides as powers of 2: $4 = 2^2$
Equate the powers: 4x + 4 = 2x + 4
2x = 0, so x = 0
**Answer: x = 0**