Day 136 **IGCSE 10-a-day: Model Solutions**

1 Turning points of a cubic

A curve has equation $y = -2x^3 + 6x^2 - 3$.
Find the coordinates of the turning points of the curve. Use the second derivative to determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 + 12x = -6x(x - 2)$

$\frac{dy}{dx} = 0$ gives $x = 0$ or $x = 2$

$y = -3$ when $x = 0$ and $y = 5$ when $x = 2$

$\frac{d^2y}{dx^2} = -12x + 12$: at $x = 0$ it is 12 (minimum), at $x = 2$ it is -12 (maximum)

Answer: (0, −3) minimum, (2, 5) maximum

2 Vector between two points

A is the point (1, 1) and B is the point (1, 0).
(a) Write down the vector **AB**.
(b) Find |**AB**|, giving the answer as a surd in simplest form if it is not exact.

(a) $\mathbf{AB} = \begin{pmatrix}1\\0\end{pmatrix} - \begin{pmatrix}1\\1\end{pmatrix} = \begin{pmatrix}0\\-1\end{pmatrix}$

(b) $|\mathbf{AB}| = \sqrt{(0)^2 + (-1)^2} = \sqrt{1} = 1$

Answer: (a) $\begin{pmatrix}\mathbf{0}\\\mathbf{-1}\end{pmatrix}$ **(b) 1**

3 Tangent at a turning point

The graph of $y = -x^2 - 8x + 7$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -2x - 8$

At P, $\frac{dy}{dx} = 0$, so $-2x - 8 = 0$ and $x = -4$

$y = 23$ at P

The gradient is 0, so the tangent is horizontal: $y = 23$

Answer: y = 23

4 Sine rule: finding a side

Work out the length of AC.
Give your answer correct to 3 significant figures. (Calculator)

$\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$AC = \frac{12.7 \times \sin 65°}{\sin 74°}$

$= 12.0$

Answer: 12.0 cm

5 Chords and Pythagoras in circles

AB is a chord of a circle, centre O. M is the midpoint of AB. (Calculator)
(a) Work out the radius of the circle.
(b) Work out angle AOB, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AM = 48 \div 2 = 24$ cm

$OA^2 = OM^2 + AM^2 = 7^2 + 24^2 = 625$

$OA = 25$ cm

(b) $\sin(\text{angle AOM}) = \frac{24}{25}$

Angle AOM = 73.74°, so angle AOB = 2 × 73.74 = 147.5°

Answer: (a) 25 cm (b) 147.5°

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6 Quadratic graphs

The curve $y = -x^2 + 14x - 33$ crosses the x-axis at the points A and B.
(a) Find the coordinates of A and B.
(b) Find the coordinates of the maximum point of the curve.

(a) Put $y = 0$. $-x^2 + 14x - 33 = 0$, so $-(x^2 + 14x + 33) = 0$, giving $-(x - 3)(x - 11) = 0$
$x - 3 = 0$ or $x - 11 = 0$, so $x = 3$ or $x = 11$
(b) The turning point is halfway between A and B: $x = (3 + 11) \div 2 = 7$
When $x = 7$: $y = -1 \times 7^2 + 14 \times 7 - 33 = 16$
Answer: (a) A(3, 0), B(11, 0) (b) (7, 16)

7 Stationary points of a quartic

A curve has equation $y = x^4 - 2x^2 + 1$.
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 4x = 4x(x - 1)(x + 1)$
$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 1$ or $x = -1$
$y = 1$ when $x = 0$, and $y = 0$ when $x = \pm 1$
$\frac{d^2y}{dx^2} = 12x^2 - 4$: it is -4 at $x = 0$ (maximum) and 8 at $x = \pm 1$ (minimum)
Answer: (0, 1) maximum; (1, 0) and (−1, 0) minima

8 HCF and LCM

The HCF of two numbers is 9 and their LCM is 396.
One of the numbers is 99.
Work out the other number.

HCF × LCM = product of the two numbers
$9 \times 396 = 3564$
Other number $= 3564 \div 99 = 36$
Answer: 36

9 Circle and line intersections

Solve the simultaneous equations
$x^2 + y^2 = 5$
$y = x - 1$

Substitute into the first equation: $x^2 + (x - 1)^2 = 5$
Expand: $2x^2 - 2x - 4 = 0$
Divide by 2: $x^2 - x - 2 = 0$
Factorise: $(x + 1)(x - 2) = 0$, so $x = -1$ or $x = 2$
$y = x - 1$ gives $y = -2$ and $y = 1$
Answer: $x = -1$, $y = -2$ and $x = 2$, $y = 1$

10 Exponential growth and decay

A population of fish starts at 800 and grows by 20% each year.
After how many whole years will the population first be at least double its starting value? (Calculator)

Multiplier = 1.2, double is 1,600
After 3 years: $800 \times 1.2^3 = 1382.4$
After 4 years: $800 \times 1.2^4 = 1658.9$
Answer: 4 years