Day 135 **IGCSE 10-a-day: Model Solutions**

1 Trapezium with vectors

OABC is a trapezium. OA is parallel to CB and CB = 2 × OA.
$\mathbf{OA} = \mathbf{a}$ and $\mathbf{OC} = \mathbf{c}$.
(a) Find $\mathbf{AB}$ in terms of $\mathbf{a}$ and $\mathbf{c}$.
(b) AM : MB = 1 : 3. Find $\mathbf{OM}$.

(a) $\mathbf{CB} = 2\mathbf{a}$
$\mathbf{OB} = \mathbf{OC} + \mathbf{CB} = 2\mathbf{a} + \mathbf{c}$
$\mathbf{AB} = -\mathbf{OA} + \mathbf{OB} = \mathbf{a} + \mathbf{c}$

(b) $\mathbf{OM} = \mathbf{OA} + \frac{1}{4}\mathbf{AB} = \mathbf{a} + \frac{1}{4}(\mathbf{a} + \mathbf{c})$

$= \frac{5}{4}\mathbf{a} + \frac{1}{4}\mathbf{c}$

Answer: (a) $\mathbf{a} + \mathbf{c}$ (b) $\frac{5}{4}\mathbf{a} + \frac{1}{4}\mathbf{c}$

2 Inequalities

(a) Solve $-4 - 5x \leq 16$
(b) n is an integer. Write down all the values of n that satisfy
$-9 \leq 2n - 1 \leq -3$

(a) Add 4: $-5x \leq 20$
Divide by -5 and reverse the inequality sign: $x \geq -4$
(b) Add 1: $-8 \leq 2n \leq -2$
Divide by 2: $-4 \leq n \leq -1$
Integers: $-4, -3, -2, -1$
Answer: (a) $x \geq -4$ (b) $-4, -3, -2, -1$

3 Percentages

A is 60% less than D.
D is 30% more than C.
Work out A as a percentage of C.

A = 0.4 × D and D = 1.3 × C
A = 0.4 × 1.3 × C = 0.52 × C
0.52 × 100 = 52%
Answer: 52%

4 Nature from the gradient either side

$y = 2x^3 - 6x - 1$
(a) Show that there is a stationary point at x = 1.
(b) By finding the gradient when $x = \frac{1}{2}$ and when $x = \frac{3}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = 6x^2 - 6$, so when x = 1: $\frac{dy}{dx} = 0$

$x = \frac{1}{2}$: gradient $= -\frac{9}{2}$ (negative)

$x = \frac{3}{2}$: gradient $= \frac{15}{2}$ (positive)

The gradient goes from negative to 0 to positive, so $(1, -5)$ is a minimum
Answer: (b) minimum

5 Equations with indices

Solve $5^{x-2} = 25$

Write the right-hand side as a power of 5: $25 = 5^2$
Equate the powers: $x - 2 = 2$
$x = 4$
Answer: $x = 4$

6 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 6) cm, DE = 4 cm and BC = 10 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$\frac{x}{2x + 6} = \frac{4}{10}$
$10x = 4(2x + 6)$
$10x = 8x + 24$
$2x = 24$
$x = 12$
(b) $AB = 2x + 6 = 30$ cm
Answer: (a) x = 12 (b) 30 cm

7 Transformed curve from a diagram

The diagram shows the graph of y = f(x) with a minimum point at (0, 1).
The dashed curve C is the image of y = f(x) after a translation.
The minimum point of C is (−2, 3).
Write down the equation of C in terms of f.

The minimum point has moved from (0, 1) to (−2, 3)
Translation by $\begin{pmatrix} -2 \\ 2 \end{pmatrix}$
x moves left by 2: f(x + 2). y moves up by 2: + 2
Answer: y = f(x + 2) + 2

8 Normal meets the curve again (quadratic)

The normal to the curve $y = x^2 - 5x + 3$ at the point P where x = 2 meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 2x - 5$: at P, gradient = −1 and y = −3
Normal: $y = x - 5$
Intersection: $x^2 - 6x + 8 = 0$
One root is x = 2, and the sum of the roots is 6
So $x_Q = 6 - 2 = 4$
y = −1
Answer: (4, −1)

9 Mixtures and percentage profit

A shop mixes two types of tea.
Type A costs £14.00 per kg and type B costs £12.00 per kg.
They are mixed in the ratio A : B = 2 : 3.
The mixture is sold for £14.08 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 2 × £14.00 + 3 × £12.00 = £64.00
Cost per kg = £64.00 ÷ 5 = £12.80
Profit per kg = £14.08 − £12.80 = £1.28
Percentage profit = 1.28 ÷ 12.80 × 100 = 10%
Answer: 10%

10 Three stationary points of a quartic

$y = 3x^4 - 24x^2 - 9$

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 12x^3 - 48x = 12x(x - 2)(x + 2)$

$\frac{dy}{dx} = 0$ gives $x = -2$, $x = 0$ or $x = 2$

Points: $(-2, -57)$, $(0, -9)$, $(2, -57)$

$\frac{d^2y}{dx^2} = 36x^2 - 48$

$x = -2$: $\frac{d^2y}{dx^2} = 96$; $x = 0$: $\frac{d^2y}{dx^2} = -48$; $x = 2$: $\frac{d^2y}{dx^2} = 96$

So $(-2, -57)$ is a minimum, $(0, -9)$ is a maximum, $(2, -57)$ is a minimum

Answer: $(-2, -57)$ minimum; $(0, -9)$ maximum; $(2, -57)$ minimum