Day 134 **IGCSE 10-a-day: Model Solutions**

1 Local maximum and minimum values

$f(x) = x^3 - 3x + 7$

Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 3x^2 - 3 = 3(x - 1)(x + 1)$

$f'(x) = 0$ gives $x = -1$ or $x = 1$

$f''(x) = 6x$: $f''(-1) = -6 < 0$ (maximum) and $f''(1) = 6 > 0$ (minimum)

$f(-1) = 9$ and $f(1) = 5$

Answer: Local maximum y = 9, local minimum y = 5

2 Inequalities

(a) Solve $2 - 4x \geq 10$

(b) n is an integer. Write down all the values of n that satisfy

$-1 < 2n - 5 \leq 9$

(a) Subtract 2: $-4x \geq 8$

Divide by -4 and reverse the inequality sign: $x \leq -2$

(b) Add 5: $4 < 2n \leq 14$

Divide by 2: $2 < n \leq 7$

Integers: 3, 4, 5, 6, 7

Answer: (a) $x \leq -2$ (b) 3, 4, 5, 6, 7

3 nth term of a quadratic sequence

Here are the first five terms of a quadratic sequence.

$-2, -3, -6, -11, -18$

Find an expression for the nth term.

First differences: $-1, -3, -5, -7$. Second differences: $-2, -2, -2$

Second difference = 2a, so $a = -1$

Subtract $-n^2$: $-1, 1, 3, 5, 7$

This is linear: $2n - 3$

nth term = $-n^2 + 2n - 3$

Answer: $-n^2 + 2n - 3$

4 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -3t^2 + 18t + 2$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 8. State whether it is moving up or down.

$\frac{dh}{dt} = -6t + 18 = 0$ gives $t = 3$

$\frac{d^2h}{dt^2} = -6 < 0$ so maximum. $h = 29$

When $t = 8$: $\frac{dh}{dt} = 18 - 6 \times 8 = -30$

Answer: (a) 29 m (b) −30 m/s, moving down

5 Average speed

A cyclist travels the first half of a journey at 60 km/h.

It travels the second half of the same journey at 90 km/h.

Work out the average speed for the whole journey.

Let each half of the journey be 180 km (a common multiple of 60 and 90)

Time for first half = 180 ÷ 60 = 3 hours

Time for second half = 180 ÷ 90 = 2 hours

Average speed = total distance ÷ total time = 360 ÷ 5 = 72 km/h

Answer: 72 km/h

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6 Solve f′(x) = 0 with a reciprocal term

$f(x) = 2x^3 + \frac{3750}{x}$

Find the values of x for which f′(x) = 0.

$f'(x) = 6x^2 - \frac{3750}{x^2}$

$f'(x) = 0$: $6x^2 = \frac{3750}{x^2}$

$x^4 = 625$

$x = 5$ or $x = -5$

Answer: $x = 5$ or $x = -5$

7 Surds

Given that $5\sqrt{3} = \sqrt{n}$,
find the value of n.

$5\sqrt{3} = \sqrt{25} \times \sqrt{3} = \sqrt{75}$

$n = 75$

Answer: $n = 75$

8 Linear programming in context

A craft shop makes x necklaces and y bracelets each day.
Each necklace needs 4 minutes of making time and 5 beads. Each bracelet needs 1 minute of making time and 5 beads.
There are 44 minutes of making time and 70 beads available.
The profit is £3 for each necklace and £2 for each bracelet.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $4x + y \leq 44$ and $5x + 5y \leq 70$

(b) Vertices of the feasible region: (0, 0), (0, 14), (10, 4), (11, 0)

Profit = $3x + 2y$: 0, 28, 38, 33

Greatest profit = £38 with $x = 10$, $y = 4$

Answer: (a) $4x + y \leq 44$, $5x + 5y \leq 70$ (b) £38

9 Vector proof: collinear points

OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
OP = $-\mathbf{a} + 3\mathbf{b}$, OQ = $2\mathbf{a} + 6\mathbf{b}$ and OR = $3\mathbf{a} + 7\mathbf{b}$.
(a) Show that P, Q and R lie on a straight line.
(b) Work out the ratio PQ : QR in its simplest form.

(a) PQ = OQ − OP = $3\mathbf{a} + 3\mathbf{b}$

QR = OR − OQ = $\mathbf{a} + \mathbf{b}$

QR = $\frac{1}{3}$ PQ, so PQ and QR are parallel, and they share the point Q

So P, Q and R lie on a straight line

(b) PQ : QR = 3 : 1

Answer: (b) 3 : 1

10 Percentage change in area and volume

The side length of a square is increased by 20%.
Work out the percentage change in the area of the square.

Side length multiplier = 1.2

Area multiplier = $1.2^2 = 1.44$

1.44 − 1 = 0.44, so a 44% increase

Answer: 44% increase