Day 132 **IGCSE 10-a-day: Model Solutions**

1 Fraction problems

A bag contains counters.

$\frac{1}{5}$ of the counters are red and $\frac{1}{4}$ of the counters are blue.

The rest of the counters, 55 of them, are green.

How many counters are in the bag?

Fraction that are green = $1 - \frac{1}{5} - \frac{1}{4} = \frac{11}{20}$

$\frac{11}{20}$ of the total is 55

Total = $55 \div \frac{11}{20} = 100$

Answer: 100

2 Areas of similar triangles

In the diagram, DE is parallel to BC. AD = 6 cm, DB = 9 cm and the area of triangle ADE is 72 cm^2.

Work out the area of the trapezium DBCE.

The diagram is not drawn accurately.

Triangles ADE and ABC are similar. AB = 6 + 9 = 15 cm

Length scale factor = $15 \div 6 = \frac{5}{2}$

Area scale factor = $(\frac{5}{2})^2 = \frac{25}{4}$

Area of triangle ABC = $72 \times \frac{25}{4} = 450$ cm^2

Trapezium = $450 - 72 = 378$ cm^2

Answer: 378 cm^2

3 Upper and lower bounds

a = 0.78 and b = 0.16,

both correct to 2 decimal places.

Work out the upper bound of a − b.

Each value is rounded, so it can be up to 0.005 above or below

$0.775 \leq a < 0.785$; $0.155 \leq b < 0.165$

Biggest answer: biggest a minus smallest b = 0.785 − 0.155 = 0.63

Answer: 0.63

4 Iteration: rearranging equations

(a) Show that the equation $x^3 - x - 20 = 0$ has a solution between x = 2 and x = 3.

(b) Show that the equation can be rearranged to give $x = \sqrt[3]{x + 20}$

(a) Let $f(x) = x^3 - x - 20$. $f(2) = 2^3 - 2 - 20 = -14$ and $f(3) = 3^3 - 3 - 20 = 4$

f(2) is negative and f(3) is positive. There is a change of sign, so there is a solution between 2 and 3

(b) Rearrange $x^3 - x - 20 = 0$: $x^3 = x + 20$

Take the cube root of both sides: $x = \sqrt[3]{x + 20}$

Answer: Shown

5 Curves that touch

The curves $y = -x^2 + 5x - 2$ and $y = -2x^2 + 9x - 6$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $x^2 - 4x + 4 = 0$
That is $(x - 2)^2 = 0$ with a repeated root $x = 2$
$y = 4$, so the point is (2, 4)
$\frac{dy}{dx} = -2x + 5$ on the first curve: gradient 1
Tangent: $y = x + 2$
Answer: (2, 4), y = x + 2

6 Compound interest: find the rate

£2,500 is invested for 3 years at r% per year compound interest.
At the end of 3 years the investment is worth £3,327.50.
Work out the value of r. (Calculator)

$2500 \times (1 + \frac{r}{100})^3 = £3{,}327.50$

$(1 + \frac{r}{100})^3 = 3327.5 \div 2500 = 1.331$

$1 + \frac{r}{100} = 1.1$
$r = 10$
Answer: r = 10

7 At least one

The probability that the 8:15 train is late on any day is $\frac{1}{5}$.

Events on different occasions are independent.
Work out the probability that this happens at least once in 6 days.
Give your answer correct to 3 significant figures. (Calculator)

$P(\text{never}) = (1 - \frac{1}{5})^6 = \frac{4}{5}^6 = 0.262144$
$P(\text{at least once}) = 1 - P(\text{never}) = 1 - 0.262144$
$= 0.738$
Answer: 0.738

8 Minimum distance with 1/t

A particle P moves on a straight line. At time t seconds (t > 0), its distance from a point O is s metres, where
$s = 2t^2 + \frac{256}{t}$
Show that P is at its minimum distance from O when t = 4, and find this minimum distance.

$s = 2t^2 + 256t^{-1}$, so $v = \frac{ds}{dt} = 4t - 256t^{-2}$

$v = 0$: $4t = \frac{256}{t^2}$, so $t^3 = 64$ and $t = 4$

$\frac{d^2s}{dt^2} = 4 + \frac{512}{t^3}$, which is 12 when t = 4, so a minimum
$s = 96$
Answer: 96 m

9 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.

$u_1 = 8$ and $u_2 = x$.

(a) Show that $u_5 = 3x + 16$.

(b) Given that $u_5 = 28$, find x and hence work out u_8.

(a) $u_3 = 8 + x$, $u_4 = u_3 + u_2 = 2x + 8$, $u_5 = u_4 + u_3 = 3x + 16$

(b) $3x + 16 = 28$ so $3x = 12$ and $x = 4$

Terms: 8, 4, 12, 16, 28, 44, 72, 116

$u_8 = 116$

Answer: $x = 4$, $u_8 = 116$

10 Expand then differentiate

$y = (3x - 2)^2$

Find $\frac{dy}{dx}$.

Expand: $y = 9x^2 - 12x + 4$

$9x^2 \rightarrow 18x$, $-12x \rightarrow -12$

$\frac{dy}{dx} = 18x - 12$

Answer: $\frac{dy}{dx} = 18x - 12$