Day 131 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(a - b)^2 - (a + b)^2$
when $a = -1$, $b = -4$ and $c = -2$.

Substitute the values: $(-1 - (-4))^2 - (-1 + (-4))^2$
$= 9 - 25$
$= -16$
Answer: -16

2 Gradient at a point

Work out the gradient of the curve $y = -2x^3 + 4x^2 + 5x$ at the point where $x = 0$.

$\frac{dy}{dx} = -6x^2 + 8x + 5$
When $x = 0$: gradient $= 5$
Answer: 5

3 Area and quadratic equations

The diagram shows a rectangle of length $(2x + 4)$ cm, width $(2x + 2)$ cm and area 360 cm^2.
(a) Show that $4x^2 + 12x - 352 = 0$.
(b) Work out the perimeter of the rectangle.

(a) $(2x + 4)(2x + 2) = 360$
Expand: $4x^2 + 12x + 8 = 360$, so $4x^2 + 12x - 352 = 0$
(b) Factorise: $4(x - 8)(x + 11) = 0$
$x = 8$ ($x = -11$ is not possible as a length would be negative)
Length = 20 cm and width = 18 cm
Perimeter $= 2 \times (20 + 18) = 76$ cm
Answer: (b) 76 cm

4 Arithmetic sequence from two terms

An arithmetic sequence has term 4 equal to 23 and term 7 equal to 29.
(a) Find an expression for the nth term.
(b) Find the first term of the sequence that is greater than 300.

(a) Common difference $= (29 - 23) \div 3 = 2$
nth term $= 2n + c$; $23 = 2 \times 4 + c$ so $c = 15$
nth term $= 2n + 15$
(b) $2n + 15 > 300$: $n > 142.5$
Smallest whole number $n = 143$; the term is 301
Answer: (a) $2n + 15$ (b) 301

5 Fractional power functions

$f(x) = -6x^{5/2} - 6x^{3/2}$
Find $f'(x)$ and work out the value of $f'(1)$.

$-6x^{5/2} \to -15x^{3/2}$, $-6x^{3/2} \to -9x^{1/2}$
$f'(x) = -15x^{3/2} - 9x^{1/2}$
$f'(1)$: $\sqrt{1} = 1$, so $f'(1) = -24$
Answer: $f'(x) = -15x^{3/2} - 9x^{1/2}$, $f'(1) = -24$

6 Cosine rule: finding an angle

Work out the size of angle B.
Give your answer correct to 1 decimal place. (Calculator)

$\cos B = \frac{12.1^2 + 4^2 - 9.7^2}{2 \times 12.1 \times 4}$
$\cos B = 0.7058$
$B = 45.1°$
Answer: 45.1°

7 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 + 2x^2 + 3x - 6$ at the point P where $x = -1$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 + 4x + 3$, so at P the gradient is 2 and $y = -8$

Tangent: $y = 2x - 6$

Intersection: $x^3 + 2x^2 + x = 0$

$x = -1$ is a repeated root, so the roots are -1, -1 and x_Q

Sum of roots $= -2$, so $-2 + x_Q = -2$ and $x_Q = 0$

$y = -6$

Answer: (0, −6)

8 Exact lengths and areas

A rectangle has area $(14 + 7\sqrt{2})$ cm^2 and length $(3 + \sqrt{2})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{2}$

Width = area ÷ length = $\frac{14 + 7\sqrt{2}}{3 + \sqrt{2}}$

Multiply top and bottom by $(3 - \sqrt{2})$:

Numerator: $(14 + 7\sqrt{2})(3 - \sqrt{2}) = 28 + 7\sqrt{2}$

Denominator: $9 - 2 = 7$

Width $= 4 + \sqrt{2}$

Answer: $4 + \sqrt{2}$ cm

9 Finding the rate of interest

Nadia invested £2,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £2,809.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $2500 \times m^2 = 2809.00$

$m^2 = 2809.00 \div 2500 = 1.1236$

$m = \sqrt{1.1236} = 1.06$

Rate $= (1.06 - 1) \times 100 = 6\%$

Answer: 6% per year

10 Bounds of roots

The volume of a cube is 689 cm^3, correct to the nearest cm^3.
Work out the upper bound of the edge length of the cube.
Give your answer correct to 3 significant figures. (Calculator)

$688.5 \leq$ volume < 689.5

Upper bound of edge $= \sqrt[3]{689.5}$

$= 8.83442... = 8.83$ (3 s.f.)

Answer: 8.83 cm