Day 130 **IGCSE 10-a-day: Model Solutions**

1 Rationalising the denominator

Write $\frac{3}{3+\sqrt{5}}$ in the form $\frac{p+q\sqrt{5}}{r}$
where p, q and r are integers and r > 1,
and the fraction is in its simplest form.

Multiply top and bottom by $(3-\sqrt{5})$:

$$\frac{3}{3+\sqrt{5}} = \frac{3(3-\sqrt{5})}{(3+\sqrt{5})(3-\sqrt{5})}$$

Denominator: $9-5=4$

$$= \frac{9-3\sqrt{5}}{4}$$

Answer: $\mathbf{\frac{9-3\sqrt{5}}{4}}$

2 Integers satisfying a double inequality

n is an integer.
Work out the sum of all the values of n that satisfy
$-2 < -n - 1 \leq 5$

Add 1 from each part: $-1 < -n \leq 6$
Divide each part by -1 (reverse the signs): $-6 \leq n < 1$
Integers: $-6, -5, -4, -3, -2, -1, 0$
Sum = -21
Answer: -21

3 Similar triangles with algebra

In the diagram, DE is parallel to BC.
AD = x cm, DB = (x + 3) cm, DE = 9 cm and BC = 21 cm.
(a) Find the value of x.
(b) Work out the length of AB.
The diagram is not drawn accurately.

(a) Triangles ADE and ABC are similar, so $\frac{AD}{AB} = \frac{DE}{BC}$

$$\frac{x}{2x+3} = \frac{9}{21}$$

$21x = 9(2x + 3)$
$21x = 18x + 27$
$3x = 27$
$x = 9$
(b) AB = 2x + 3 = 21 cm
Answer: (a) x = 9 (b) 21 cm

4 Increasing for x greater than k

The function $f(x) = -x^2 + 8x - 3$ is decreasing when x > k.
Find the value of k.

$f'(x) = -2x + 8$
The turning point is where $f'(x) = 0$: $-2x + 8 = 0$
$x = 4$
$f'(x)$ is negative for x > 4, so k = 4
Answer: k = 4

5 Conditional probability from a table

The table shows information about 110 students.
(a) A student who studies French is chosen at random. Work out the probability that this student is a boy.
(b) Two of the 110 students are chosen at random. Work out the probability that both are boys.

(a) 54 students are in the French column, and 25 of them are boys

Probability = $\frac{25}{54}$

(b) There are 52 boys, so P(both) = $\frac{52}{110} \times \frac{51}{109} = \frac{1326}{5995}$

Answer: (a) $\frac{25}{54}$ (b) $\frac{1326}{5995}$

6 Equations in disguise

Solve $x^4 - 61x^2 + 900 = 0$

Let $u = x^2$: $u^2 - 61u + 900 = 0$
Factorise: $(u - 36)(u - 25) = 0$, so $u = 36$ or $u = 25$
$x^2 = 36$ or $x^2 = 25$
$x = \pm 6$ or $x = \pm 5$
Answer: $x = -6, -5, 5, 6$

7 Nature from the gradient either side

$y = -2x^3 - 9x^2 - 6$
(a) Show that there is a stationary point at $x = 0$.
(b) By finding the gradient when $x = -\frac{1}{2}$ and when $x = \frac{1}{2}$, decide whether it is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 - 18x$, so when $x = 0$: $\frac{dy}{dx} = 0$

$x = -\frac{1}{2}$: gradient = $\frac{15}{2}$ (positive)

$x = \frac{1}{2}$: gradient = $-\frac{21}{2}$ (negative)

The gradient goes from positive to 0 to negative, so $(0, -6)$ is a maximum
Answer: (b) maximum

8 Transformations of graphs

The graph of $y = f(x)$ has a maximum point at $(-4, -2)$.
Write down the coordinates of the turning point of the graph of
(a) $y = f(x + 1)$
(b) $y = 3f(x + 2)$

(a) Start with the turning point $(-4, -2)$
Apply the transformation to the point: $(-5, -2)$
(b) $f(x + 2)$ changes the x-coordinate to -6, and $3f(x)$ changes the y-coordinate to -6
Turning point: $(-6, -6)$
Answer: (a) $(-5, -2)$ (b) $(-6, -6)$

9 Tangent meets the curve again (cubic)

The tangent to the curve $y = x^3 - 2x^2 + 3x - 2$ at the point P where $x = 2$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = 3x^2 - 4x + 3$, so at P the gradient is 7 and $y = 4$

Tangent: $y = 7x - 10$
Intersection: $x^3 - 2x^2 - 4x + 8 = 0$
$x = 2$ is a repeated root, so the roots are 2, 2 and x_Q
Sum of roots = 2, so $4 + x_Q = 2$ and $x_Q = -2$
$y = -24$
Answer: $(-2, -24)$

10 Linear programming in context

A bakery makes x cakes and y loaves in a morning.
Each cake needs 6 minutes of oven time and 2 kg of flour. Each loaf needs 2 minutes of oven time and 2 kg of flour.
There are 72 minutes of oven time and 32 kg of flour available.
The profit is £8 for each cake and £7 for each loaf.
(a) Write down two inequalities, other than $x \geq 0$, $y \geq 0$. (b) Find the greatest profit.

(a) $6x + 2y \leq 72$ and $2x + 2y \leq 32$
(b) Vertices of the feasible region: (0, 0), (0, 16), (10, 6), (12, 0)
Profit = 8x + 7y: 0, 112, 122, 96
Greatest profit = £122 with x = 10, y = 6
Answer: (a) $6x + 2y \leq 72$, $2x + 2y \leq 32$ (b) £122