Day 13 **IGCSE 10-a-day: Model Solutions**

1 Expand then differentiate

$y = x(x - 3)(3x - 1)$

Find $\frac{dy}{dx}$.

Expand: $y = 3x^3 - 10x^2 + 3x$

$3x^3 \to 9x^2$, $-10x^2 \to -20x$, $3x \to 3$

$\frac{dy}{dx} = 9x^2 - 20x + 3$

Answer: $\mathbf{\frac{dy}{dx} = 9x^2 - 20x + 3}$

2 Angles with algebra

ABC is an isosceles triangle with AB = AC.
Angle BAC = $(5x - 20)°$
Angle ABC = $(2x + 28)°$
(a) Work out the value of x.
(b) Work out the size of angle BAC.

(a) Angles ABC and ACB are equal, and the angles in a triangle add up to 180°
$(5x - 20) + 2(2x + 28) = 180$
$9x + 36 = 180$, so $9x = 144$
$x = 16$
(b) Angle BAC = $5 \times 16 - 20 = 60°$
Answer: (a) x = 16 (b) 60°

3 Inverse of a linear function

$f(x) = -2x + 2$.
(a) Find $f^{-1}(x)$.
(b) Find $f^{-1}(8)$.

(a) Let $y = -2x + 2$, then rearrange to make x the subject
$x = \frac{2 - y}{2}$, so $f^{-1}(x) = \frac{2 - x}{2}$

(b) $f^{-1}(8) = -3$

Answer: (a) $\mathbf{f^{-1}(x) = \frac{2 - x}{2}}$ **(b)** $\mathbf{-3}$

4 Distance-time graphs

The graph shows Hassan's cycle ride to a café and back.
(a) Work out Hassan's speed on the way back.
(b) Work out Hassan's average speed for the whole ride, including the stop.

(a) Return: 15 km in 0.5 hours, so speed = 15 ÷ 0.5 = 30 km/h
(b) Total distance = 2 × 15 = 30 km, total time = 2.5 hours
Average speed = 30 ÷ 2.5 = 12 km/h
Answer: (a) 30 km/h (b) 12 km/h

5 Coordinates and gradient at the y-intercept

$y = 2x^4 + 5x^3 + 5x^2 + 2$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = 2, so A is (0, 2)
$\frac{dy}{dx} = 8x^3 + 15x^2 + 10x$

When x = 0, $\frac{dy}{dx} = 0$

Answer: A = (0, 2), gradient 0

6 Line and curve intersections

Solve the simultaneous equations

$y = x^2 - x + 4$

$y = -3x + 12$

Substitute: $x^2 - x + 4 = -3x + 12$

Rearrange: $x^2 + 2x - 8 = 0$

Factorise: $(x - 2)(x + 4) = 0$, so $x = 2$ or $x = -4$

When $x = 2$: $y = -3 \times 2 + 12 = 6$

When $x = -4$: $y = -3 \times (-4) + 12 = 24$

Answer: $x = -4$, $y = 24$ and $x = 2$, $y = 6$

7 Solving trigonometric equations

Solve $4 \tan x - 4 = 0$ for $0 \leq x \leq 360$.

$\tan x = 1$

Principal value: $x = \tan^{-1}(1) = 45°$

Other solutions in the range: use symmetry of the tan graph

$x = 45, 225$

Answer: $x = 45, 225$

8 Minimum running cost

The cost, C pence per kilometre, of running a coach at a steady speed of v km/h is modelled by

$C = \frac{1}{10}v + \frac{360}{v}$ $(v > 0)$

Find the speed at which the cost per kilometre is least, and show that it is a minimum.

Work out the least cost per kilometre.

$\frac{dC}{dv} = \frac{1}{10} - 360v^{-2} = 0$, so $v^2 = 3600$ and $v = 60$

$\frac{d^2C}{dv^2} = 720v^{-3} = \frac{1}{300} > 0$, so a minimum

$C = \frac{360}{60} + \frac{60}{10} = 12$

Answer: $v = 60$ km/h, $C = 12$ pence per km

9 Alternate segment theorem

PAQ is a tangent to the circle at A. B and C are points on the circle. Angle PAB = 50° and angle CAQ = 73°.

(a) Work out the size of angle ABC.

(b) Work out the size of angle ACB.

The diagram is not drawn accurately.

(a) Angle ABC = angle CAQ = 73° (alternate segment theorem)

(b) Angle ACB = angle PAB = 50° (alternate segment theorem)

Answer: (a) 73° (b) 50°

10 Growth and decay

A culture has 1000 bacteria.

The number of bacteria increases by 3% each hour.

After how many whole hours will there first be more than 1500 bacteria? (Calculator)

Multiplier each hour = 1.03

After 13 hours: $1000 \times 1.03^{13} = 1469$

After 14 hours: $1000 \times 1.03^{14} = 1513$

So the first time it is more than 1500 is after 14 hours

Answer: 14 hours