**Day 129** **IGCSE 10-a-day: Model Solutions**

## 1 Solving quadratics by factorising

Solve $12x^2 + 25x = 7$

Rearrange so that one side is 0: $12x^2 + 25x - 7 = 0$
Factorise: $(3x + 7)(4x - 1) = 0$
$3x + 7 = 0$ or $4x - 1 = 0$
$x = -\frac{7}{3}$ or $x = \frac{1}{4}$

**Answer: $x = -\frac{7}{3}$ or $x = \frac{1}{4}$**

## 2 Standard form: adding and subtracting

Work out $4.9 \times 10^0 + 2.2 \times 10^{-2}$.
Give your answer in standard form.

Write both numbers with the same power of 10, $10^{-2}$:
$490 \times 10^{-2} + 2.2 \times 10^{-2}$
$= (490 + 2.2) \times 10^{-2} = 492.2 \times 10^{-2}$
$= 4.922 \times 10^0$

**Answer: $4.922 \times 10^0$**

## 3 Second derivative

$y = x^4 - \frac{5}{x}$

(a) Find $\frac{dy}{dx}$ and $\frac{d^2y}{dx^2}$.

(b) Work out the value of $\frac{d^2y}{dx^2}$ when $x = 3$.

$\frac{dy}{dx} = 4x^3 + 5x^{-2}$

$\frac{d^2y}{dx^2} = 12x^2 - 10x^{-3}$

$x = 3$: $\frac{d^2y}{dx^2} = \frac{2906}{27}$

**Answer: (a) $\frac{dy}{dx} = 4x^3 + \frac{5}{x^2}$, $\frac{d^2y}{dx^2} = 12x^2 - \frac{10}{x^3}$ (b) $\frac{2906}{27}$**

## 4 Inequalities

(a) Solve $7 - 2x > -11$
(b) n is an integer. Write down all the values of n that satisfy
$3 \leq 4n - 5 \leq 15$

(a) Subtract 7: $-2x > -18$
Divide by $-2$ and reverse the inequality sign: $x < 9$
(b) Add 5: $8 \leq 4n \leq 20$
Divide by 4: $2 \leq n \leq 5$
Integers: 2, 3, 4, 5
**Answer: (a) $x < 9$ (b) 2, 3, 4, 5**

## 5 Square roots as powers

$y = -4x^{5/2} + 8x^{3/2} + 3x$

Find $\frac{dy}{dx}$. Give each term of your answer in the form $ax^n$.

Write as powers of x: $y = -4x^{5/2} + 8x^{3/2} + 3x$

$-4x^{5/2} \rightarrow -10x^{3/2}$, $8x^{3/2} \rightarrow 12x^{1/2}$, $3x \rightarrow 3$

$\frac{dy}{dx} = -10x^{3/2} + 12x^{1/2} + 3$

**Answer:** $\mathbf{\frac{dy}{dx} = -10x^{3/2} + 12x^{1/2} + 3}$

## 6 Histograms

The histogram shows the lengths of leaves collected from a tree.
(a) Work out the total number of leaves.
(b) Estimate the number of leaves with a length between 15 cm and 35 cm.

Frequency = frequency density × class width
0 to 10: 0.6 × 10 = 6
10 to 20: 1.2 × 10 = 12
20 to 30: 2 × 10 = 20
30 to 50: 0.4 × 20 = 8
(a) Total = 46
(b) 15 to 20: 1.2 × 5 = 6
Whole bar between: 20
30 to 35: 0.4 × 5 = 2, so estimate = 28
**Answer: (a) 46 (b) 28**

## 7 Show that a line is a tangent

The line $y = -8x + 10$ meets the curve $y = -2x^2 - 4x + 8$.
Show that the line is a tangent to the curve and find the coordinates of the point where they touch.

Equate: $-2x^2 - 4x + 8 = -8x + 10$, so $-2x^2 + 4x - 2 = 0$
Discriminant: $4^2 - 4 \times (-2) \times (-2) = 16 - 16 = 0$
One repeated root, so the line touches the curve at one point only: it is a tangent
$x = \frac{-4}{2 \times (-2)} = 1$ and $y = 2$
**Answer: (1, 2)**

## 8 Compound interest

Omar puts £5,000 in a savings account.
Compound interest is paid at 2% in year 1, 3% in year 2 and 2% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.02, 1.03, 1.02
£5,000 × 1.02 × 1.03 × 1.02
= £5,358.06
**Answer: £5,358.06**

## 9 Combined transformations

The point P(−3, −1) is reflected in the y-axis.
The image is then rotated 180° about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (3, −1)
After the second transformation: P'' = (−3, 1)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, y) → (x, −y)
The single mapping is (x, y) → (x, −y)
This is a reflection in the x-axis
**Answer: (a) (−3, 1) (b) Reflection in the x-axis**

## 10 Triangle shapes from position vectors

A = (−4, −2), B = (−9, 3) and C = (−9, −7).
Show that triangle ABC is isosceles and right-angled.

$\mathbf{AB} = \begin{pmatrix} -5 \\ 5 \end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix} -5 \\ -5 \end{pmatrix}$

$|AB|^2 = 25 + 25 = 50$ and $|AC|^2 = 25 + 25 = 50$

$|AB| = |AC| = \sqrt{50}$, so the triangle is isosceles

$\mathbf{BC} = \begin{pmatrix} 0 \\ -10 \end{pmatrix}$ so $|BC|^2 = 0 + 100 = 100$

$|AB|^2 + |AC|^2 = 50 + 50 = 100 = |BC|^2$, so the angle at A is 90° (Pythagoras)

**Answer: $|\mathbf{AB}| = |\mathbf{AC}|$ and $\mathbf{AB}^2 + \mathbf{AC}^2 = \mathbf{BC}^2$**