Day 128 **IGCSE 10-a-day: Model Solutions**

1 Solving quadratics by factorising

Solve $4x^2 - 2 = -7x$

Rearrange so that one side is 0: $4x^2 + 7x - 2 = 0$
Factorise: $(x + 2)(4x - 1) = 0$
$x + 2 = 0$ or $4x - 1 = 0$
$x = -2$ or $x = \frac{1}{4}$

Answer: $x = -2$ or $x = \frac{1}{4}$

2 Points with a given gradient (cubic)

The curve $y = x^3 + 6x^2 + 5x + 1$ has gradient -4 at two points.
Find the x coordinates of these points.

$\frac{dy}{dx} = 3x^2 + 12x + 5$
$3x^2 + 12x + 5 = -4$, so $3x^2 + 12x + 9 = 0$
Divide by 3: $x^2 + 4x + 3 = 0$
$(x + 3)(x + 1) = 0$, so $x = -3$ or $x = -1$
Answer: $x = -3$ and $x = -1$

3 Equations with fractions

Solve $\frac{x - 4}{3} + \frac{x + 2}{6} = -3$

Multiply every term by 6: $2(x - 4) + (x + 2) = -18$
Expand: $3x - 6 = -18$
$3x = -12$
$x = -4$
Answer: $x = -4$

4 Standard form: multiplying and dividing

Work out $(5.12 \times 10^0) \div (1.6 \times 10^8)$.
Give your answer in standard form.

Divide the numbers: $5.12 \div 1.6 = 3.2$
Subtract the powers: $10^0 \div 10^8 = 10^{-8}$
$3.2 \times 10^{-8} = 3.2 \times 10^{-8}$
Answer: 3.2×10^{-8}

5 Cosine rule and area of a triangle

The diagram shows triangle ABC. (Calculator)
(a) Work out the length of AB, marked x, to 1 decimal place.
(b) Work out the area of triangle ABC to 3 significant figures.
The diagram is not drawn accurately.

(a) $x^2 = 12^2 + 10^2 - 2 \times 12 \times 10 \times \cos 100°$
$x^2 = 285.68$
$x = 16.9$ cm (1 d.p.)

(b) Area $= \frac{1}{2} \times 12 \times 10 \times \sin 100° = 59.1$ cm^2 (3 s.f.)

Answer: (a) 16.9 cm (b) 59.1 cm^2

6 Standard form in context

A town uses 7.3×10^4 litres of water each day. A reservoir holds 7.2×10^6 litres.
For how many days would the reservoir supply the town? Give your answer in standard form to 3 significant figures.
(Calculator)

Days = $(7.2 \times 10^6) \div (7.3 \times 10^4)$
Divide the numbers: 9.8630, and subtract the powers of 10
$= 9.86 \times 10^1$
Answer: 9.86×10^1 days

7 Greatest speed from displacement

A particle moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = -t^3 + 15t^2 + 6t$ for $t \geq 0$
Find the greatest speed of the particle, and the value of t at which it occurs.

$v = \frac{ds}{dt} = -3t^2 + 30t + 6$

$a = \frac{dv}{dt} = -6t + 30$

Maximum v when a = 0: $-6t + 30 = 0$, so $t = 5$

$\frac{d^2v}{dt^2} = -6 < 0$, so v is a maximum

$v = 81$
Answer: 81 m/s when t = 5 s

8 Independent events and probability rules

A and B are two events.
P(A) = 0.2, P(B) = 0.5 and P(A or B) = 0.6
(a) Work out P(A and B).
(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$
$0.6 = 0.2 + 0.5 - P(A \cap B)$, so $P(A \cap B) = 0.1$
(b) $P(A) \times P(B) = 0.2 \times 0.5 = 0.1$
This equals $P(A \cap B)$, so A and B are independent
Answer: (a) 0.1 (b) Yes

9 Tangent meets the curve again

A curve has equation $y = x^3 - 4x^2 + 7x + 8$.
The tangent to the curve at the point A, where x = 0, meets the curve again at the point B.
Find the coordinates of B.

$\frac{dy}{dx} = 3x^2 - 8x + 7$, so the gradient at A is 7, and A = (0, 8)
Tangent: $y = 7x + 8$
Equate: $x^3 - 4x^2 = 0$, which has a repeated root x = 0
Sum of the roots = 4, so $0 + 0 + x = 4$ and $x = 4$
$y = 36$
Answer: B (4, 36)

10 Fractional indices

Given that $x > 0$, solve $x^{-\frac{1}{4}} = \frac{1}{3}$

Raise both sides to the power -4
$x = (\frac{1}{3})^{-4}$
$x = 81$
Answer: x = 81