Day 127 **IGCSE 10-a-day: Model Solutions**

1 Maximum speed from velocity

A particle moves in a straight line. Its velocity, v m/s, at time t seconds is given by
$v = 32t - 4t^2$ for $0 \leq t \leq 8$
Find the time at which the speed of the particle is greatest, and this greatest speed.

$a = \frac{dv}{dt} = -8t + 32$

Greatest speed when $a = 0$: $-8t + 32 = 0$, so $t = 4$

$\frac{d^2v}{dt^2} = -8 < 0$, so this is a maximum

$v = 32 \times 4 - 4 \times 4^2 = 64$

Answer: t = 4 s, greatest speed 64 m/s

2 Multiplying 2 by 2 matrices

$A = \begin{pmatrix} 0 & 5 \\ 0 & -1 \end{pmatrix}$ and $B = \begin{pmatrix} -2 & 2 \\ 4 & -4 \end{pmatrix}$.

Work out $A^2 - B^2$.

$A^2 = \begin{pmatrix} 0 & 5 \\ 0 & -1 \end{pmatrix} \times \begin{pmatrix} 0 & 5 \\ 0 & -1 \end{pmatrix} = \begin{pmatrix} 0 & -5 \\ 0 & 1 \end{pmatrix}$

$B^2 = \begin{pmatrix} -2 & 2 \\ 4 & -4 \end{pmatrix} \times \begin{pmatrix} -2 & 2 \\ 4 & -4 \end{pmatrix} = \begin{pmatrix} 12 & -12 \\ -24 & 24 \end{pmatrix}$

$A^2 - B^2 = \begin{pmatrix} -12 & 7 \\ 24 & -23 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-12} & \mathbf{7} \\ \mathbf{24} & \mathbf{-23} \end{pmatrix}$

3 Mixtures and percentage profit

A shop mixes two types of coffee.
Type A costs £12.00 per kg and type B costs £4.00 per kg.
They are mixed in the ratio A : B = 4 : 1.
The mixture is sold for £13.52 per kg.
Work out the percentage profit.

Cost of 5 kg of mixture = 4 × £12.00 + 1 × £4.00 = £52.00
Cost per kg = £52.00 ÷ 5 = £10.40
Profit per kg = £13.52 − £10.40 = £3.12
Percentage profit = 3.12 ÷ 10.40 × 100 = 30%
Answer: 30%

4 Increasing or decreasing at a point

$y = 2x^3 + 15x^2 + 36x + 2$
At the point where x = 3, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 + 30x + 36$

When $x = 3$: $\frac{dy}{dx} = 180$

180 is positive, so the curve is increasing
Answer: increasing (gradient 180)

5 Combined transformations

The point P(2, 6) is rotated 180° about the origin.
The image is then rotated 90° anticlockwise about the origin.
(a) Write down the coordinates of the final image of P.
(b) Describe fully the single transformation that is equivalent to these two transformations.

(a) After the first transformation: P' = (−2, −6)
After the second transformation: P'' = (6, −2)
(b) Follow a general point (x, y) through both transformations: (x, y) → (−x, −y) → (y, −x)
The single mapping is (x, y) → (y, −x)
This is a rotation 90° clockwise about the origin
Answer: (a) (6, −2) (b) Rotation 90° clockwise about the origin

6 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are (x − 4) cm, (x + 3) cm and (x + 5) cm.
The longest side is (x + 5) cm.
(a) Show that $x^2 - 12x = 0$
(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 4)^2 + (x + 3)^2 = (x + 5)^2$
Expand: $x^2 - 8x + 16 + x^2 + 6x + 9 = x^2 + 10x + 25$
Rearrange: $x^2 - 12x = 0$
(b) Factorise: $(x - 12)x = 0$, so $x = 12$ (the other root, $x = 0$, does not give valid side lengths)
The shorter sides are 8 cm and 15 cm, so area = $\frac{1}{2} \times 8 \times 15 = 60$ cm^2
Answer: (b) 60 cm^2

7 Three stationary points of a quartic

$y = 2x^4 - 16x^2 - 7$
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 32x = 8x(x - 2)(x + 2)$
$\frac{dy}{dx} = 0$ gives $x = -2$, $x = 0$ or $x = 2$
Points: (−2, −39), (0, −7), (2, −39)
$\frac{d^2y}{dx^2} = 24x^2 - 32$
$x = -2$: $\frac{d^2y}{dx^2} = 64$; $x = 0$: $\frac{d^2y}{dx^2} = -32$; $x = 2$: $\frac{d^2y}{dx^2} = 64$
So (−2, −39) is a minimum, (0, −7) is a maximum, (2, −39) is a minimum
Answer: (−2, −39) minimum; (0, −7) maximum; (2, −39) minimum

8 Radius from chord and height of arc

A circular arc has a chord AB of length 42 cm.
The greatest distance between the chord and the arc is 7 cm.
Work out the radius of the circle.

Let the radius be r. The centre O is on the line through the midpoint M of AB and the highest point of the arc, so OM = r − 7
AM = 42 ÷ 2 = 21. Pythagoras in triangle OMA: $r^2 = (r - 7)^2 + 21^2$
$r^2 = r^2 - 14r + 49 + 441$
$14r = 490$, so $r = 35$ cm
Answer: 35 cm

9 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 12 cm.
The vertex V is vertically above the centre O of the base. VA = 20 cm.
(a) Work out the vertical height VO.
(b) Work out the angle between VA and the base. (Calculator)

(a) AC = $12\sqrt{2}$ so AO = 8.485 cm
$VO^2 = 20^2 - 8.485^2 = 328.0$
VO = 18.1 cm

(b) $\cos VAO = \frac{AO}{VA} = \frac{8.485}{20}$, angle = 64.9°

Answer: (a) 18.1 cm (b) 64.9°

10 Bounds and accuracy

a = 0.232 and b = 6.56, both correct to 3 significant figures.
Work out a × b.
Write down your answer to a suitable degree of accuracy.
You must show your working and give a reason for your answer. (Calculator)

$0.2315 \leq a < 0.2325$; $6.555 \leq b < 6.565$
Lower bound = 0.2315 × 6.555 = 1.5174825
Upper bound = 0.2325 × 6.565 = 1.5263625
Both bounds round to 1.5 to 2 significant figures, but not to 3 significant figures
Answer: 1.5 (2 s.f.)