Day 126 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x)°$, $(x + 30)°$, $(2x + 25)°$, $(2x - 35)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$8x + 20 = 540$, so $8x = 520$
$x = 65$
(b) The angles are 65°, 130°, 95°, 155°, 95°
The largest is 155°
Answer: (a) x = 65 (b) 155°

2 Standard form: multiplying and dividing

Work out $(1.5 \times 10^{-4})^2$.
Give your answer in standard form.

$(1.5 \times 10^{-4})^2 = 1.5^2 \times (10^{-4})^2$
$= 2.25 \times 10^{-8}$
$= 2.25 \times 10^{-8}$
Answer: 2.25×10^{-8}

3 Using a line of best fit

The scatter graph shows distance from town (x) and price (y).
The equation of the line of best fit is $y = -0.5x + 17$.
(a) Estimate y when x = 5.
(b) Explain why an estimate of y when x = 25 may be unreliable.

(a) $y = -0.5 \times 5 + 17 = 14.5$
(b) x = 25 is outside the range of the data on the graph, so the estimate relies on the pattern continuing (extrapolation)
Answer: (a) 14.5 (b) x = 25 is outside the range of the data

4 Vectors in a triangle

OAB is a triangle. OA = $\mathbf{a}$ and OB = $\mathbf{b}$.
M is on OA with OM : MA = 1 : 4. N is on AB with AN : NB = 4 : 1.
(a) Find ON in terms of $\mathbf{a}$ and $\mathbf{b}$.
(b) Show that MN is parallel to OB.
The diagram is not drawn accurately.

(a) AB = $-\mathbf{a} + \mathbf{b}$, so AN = $\frac{4}{5}(\mathbf{b} - \mathbf{a})$

ON = OA + AN = $\mathbf{a} + \frac{4}{5}(\mathbf{b} - \mathbf{a}) = \frac{1}{5}\mathbf{a} + \frac{4}{5}\mathbf{b}$

(b) OM = $\frac{1}{5}\mathbf{a}$

MN = ON − OM = $\frac{1}{5}\mathbf{a} + \frac{4}{5}\mathbf{b} - \frac{1}{5}\mathbf{a} = \frac{4}{5}\mathbf{b}$

MN = $\frac{4}{5}\mathbf{b}$, which is a multiple of OB, so MN is parallel to OB

Answer: (a) $\frac{1}{5}\mathbf{a} + \frac{4}{5}\mathbf{b}$ (b) MN = $\frac{4}{5}\mathbf{b}$, parallel to OB

5 Product of brackets: tangent gradient

$y = x(x - 2)(x + 2)$

Work out the gradient of the curve at the point where $x = 2$.

Expand: $y = x^3 - 4x$

$\frac{dy}{dx} = 3x^2 - 4$

$x = 2$: $\frac{dy}{dx} = 8$

Answer: 8

6 Direct and inverse proportion

V is inversely proportional to the square root of r.

When r = 16, V = 3.

(a) Find a formula for V in terms of r.

(b) Find the value of V when r = 36.

(a) $V = \frac{k}{\sqrt{r}}$

Substitute r = 16 and V = 3: $3 = \frac{k}{\sqrt{16}}$

$k = 12$

$V = \frac{12}{\sqrt{r}}$

(b) $V = \frac{12}{\sqrt{36}} = 2$

Answer: (a) $V = \frac{12}{\sqrt{r}}$ (b) 2

7 3D trigonometry: pyramid

VABCD is a pyramid. The base ABCD is a square with sides of 5 cm.

The vertex V is vertically above the centre O of the base. VA = 8 cm.

(a) Work out the vertical height VO.

(b) Work out the angle between VA and the base. (Calculator)

(a) $AC = 5\sqrt{2}$ so $AO = 3.536$ cm

$VO^2 = 8^2 - 3.536^2 = 51.50$

$VO = 7.18$ cm

(b) $\cos VAO = \frac{AO}{VA} = \frac{3.536}{8}$, angle = 63.8°

Answer: (a) 7.18 cm (b) 63.8°

8 Stationary points of a quartic

A curve has equation $y = 2x^4 - 36x^2$.

Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 8x^3 - 72x = 8x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives $x = 0$, $x = 3$ or $x = -3$

$y = 0$ when $x = 0$, and $y = -162$ when $x = \pm 3$

$\frac{d^2y}{dx^2} = 24x^2 - 72$: it is -72 at $x = 0$ (maximum) and 144 at $x = \pm 3$ (minimum)

Answer: (0, 0) maximum; (3, −162) and (−3, −162) minima

9 Rationalising the denominator

Write $\frac{2+\sqrt{7}}{4-\sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are rational numbers.

Multiply top and bottom by $(4 + \sqrt{7})$:
Numerator: $(2 + \sqrt{7})(4 + \sqrt{7}) = 15 + 6\sqrt{7}$
Denominator: $16 - 7 = 9$
$= \frac{5 + 2\sqrt{7}}{3}$

Answer: $\frac{5 + 2\sqrt{7}}{3}$

10 Normal meets the curve again (quadratic)

The normal to the curve $y = -x^2 - 4x + 1$ at the point P where $x = -1$ meets the curve again at Q.
Find the coordinates of Q.

$\frac{dy}{dx} = -2x - 4$: at P, gradient $= -2$ and $y = 4$

Normal: $y = \frac{1}{2}x + \frac{9}{2}$

Intersection: $-x^2 - \frac{9}{2}x - \frac{7}{2} = 0$

One root is $x = -1$, and the sum of the roots is $-\frac{9}{2}$

So $x_Q = -\frac{9}{2} - (-1) = -\frac{7}{2}$

$y = \frac{11}{4}$

Answer: $\left(-\frac{7}{2}, \frac{11}{4}\right)$