Day 125 **IGCSE 10-a-day: Model Solutions**

1 Comparing costs

Sparkle Cleaners has a call-out fee of £8 plus £2 per hour.
Fresh Start has a call-out fee of £13.50 plus £1.50 per hour.
(a) Work out the number of hours for which the two costs are the same.
(b) Which is cheaper for 18 hours, and by how much?

(a) Let n be the number of hours. Working in pounds, the costs are equal when $8 + 2n = 13.50 + 1.50n$
$0.50n = 5.50$, so $n = 11$
(b) Sparkle Cleaners: £8 + £2 × 18 = £44
Fresh Start: £13.50 + £1.50 × 18 = £40.50
Fresh Start is cheaper by £3.50
Answer: (a) 11 (b) Fresh Start, by £3.50

2 Stationary points and their nature

$y = -2x^3 + 3x^2 + 36x + 2$
(a) Find the coordinates of the stationary points.
(b) Use $\frac{d^2y}{dx^2}$ to decide whether each is a maximum or a minimum.

$\frac{dy}{dx} = -6x^2 + 6x + 36 = -6(x + 2)(x - 3)$

$\frac{dy}{dx} = 0$ gives $x = -2$ or $x = 3$, so the points are $(-2, -42)$ and $(3, 83)$

$\frac{d^2y}{dx^2} = -12x + 6$

$x = -2$: $\frac{d^2y}{dx^2} = 30$, which is positive, so a minimum

$x = 3$: $\frac{d^2y}{dx^2} = -30$, which is negative, so a maximum

Answer: (a) $(-2, -42)$ and $(3, 83)$ (b) $(-2, -42)$ is a minimum, $(3, 83)$ is a maximum

3 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 65°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: 2 × 65 = 130°
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = 360 − 90 − 90 − 130 = 50°
Answer: (a) 130° (b) 50°

4 Determinant and inverse

$M = \begin{pmatrix} -1 & -2 \\ 2 & -5 \end{pmatrix}$.
(a) Work out the determinant of M.
(b) Find M^{-1}.

(a) $\det M = (-1) \times (-5) - (-2) \times 2 = 9$
(b) Swap the leading diagonal, change the signs of the other two entries, divide by the determinant

$M^{-1} = \frac{1}{9}\begin{pmatrix} -5 & 2 \\ -2 & -1 \end{pmatrix}$

Answer: (a) 9 (b) $\frac{1}{9}\begin{pmatrix} -5 & 2 \\ -2 & -1 \end{pmatrix}$

5 Show that a line is a tangent

The line $y = -3x + k$ is a tangent to the curve $y = x^2 - 5x - 2$.
Find the value of the constant k.

$\frac{dy}{dx} = 2x - 5 = -3$, so $x = 1$

$y = -6$ when $x = 1$

The line passes through $(1, -6)$: $-6 = -3 \times 1 + k$, so $k = -3$

Answer: $k = -3$

6 Conditional probability from a table

The table shows information about 134 students.
(a) A student who studies French is chosen at random. Work out the probability that this student is a girl.
(b) Two of the 134 students are chosen at random. Work out the probability that both are girls.

(a) 40 students are in the French column, and 29 of them are girls

Probability = $\frac{29}{40}$

(b) There are 77 girls, so P(both) = $\frac{77}{134} \times \frac{76}{133} = \frac{22}{67}$

Answer: (a) $\frac{29}{40}$ (b) $\frac{22}{67}$

7 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = 2t^3 - 6t^2 + 8t - 5$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 12t + 8 = 6(t - 1)^2 + 2$

A square is never negative, so $v \geq 2 > 0$ and P is never at rest
Least value of v occurs when $t = 1$, giving 2
Answer: (b) 2 m/s when t = 1 s

8 Rationalising the denominator

Write $\frac{1 + \sqrt{7}}{5 - \sqrt{7}}$ in the form $p + q\sqrt{7}$
where p and q are rational numbers.

Multiply top and bottom by $(5 + \sqrt{7})$:
Numerator: $(1 + \sqrt{7})(5 + \sqrt{7}) = 12 + 6\sqrt{7}$
Denominator: $25 - 7 = 18$
$= \frac{2 + \sqrt{7}}{3}$

Answer: $\frac{2 + \sqrt{7}}{3}$

9 Probability and quadratic equations

A bag contains x black counters and 6 blue counters.
Two counters are taken at random without replacement.

The probability that both are black is $\frac{3}{8}$.

(a) Show that $5x^2 - 41x - 90 = 0$.
(b) Hence work out the number of counters in the bag.

(a) P(both black) = $\frac{x}{x+6} \times \frac{x-1}{x+5} = \frac{3}{8}$

$8\,x(x-1) = 3(x+6)(x+5)$

$8x^2 - 8x = 3x^2 + 33x + 90$ so $5x^2 - 41x - 90 = 0$

(b) Solve: $x = 10$ or $x = -\frac{9}{5}$

x cannot be negative, so x = 10
Number of counters = 10 + 6 = 16
Answer: x = 10, so 16 counters in the bag

10 Rates of work

Tap A can fill an empty tank in 24 minutes.
Tap B can fill an empty tank in 40 minutes.
Both are turned on at the same time.
How long do they take to fill an empty tank?

In 1 minute: Tap A does $\frac{1}{24}$ and Tap B does $\frac{1}{40}$

Together: $\frac{1}{24} + \frac{1}{40} = \frac{64}{960}$

Time = $\frac{960}{64}$ = 15 minutes

Answer: 15 minutes