Day 124 **IGCSE 10-a-day: Model Solutions**

1 Compound interest: how many years

£5,000 is invested at 10% per year compound interest.
Work out the least whole number of years after which the investment is worth more than £12,500. (Calculator)

Value after n years = 5000×1.1^n
After 9 years: £11,789.74 (less than £12,500)
After 10 years: £12,968.71 (more than £12,500)
Answer: 10 years

2 Fraction problems

A bag contains balloons.
$\frac{1}{6}$ of the balloons are pink and $\frac{3}{5}$ of the balloons are silver.
The rest of the balloons, 28 of them, are gold.
How many silver balloons are there?

Fraction that are gold = $1 - \frac{1}{6} - \frac{3}{5} = \frac{7}{30}$

$\frac{7}{30}$ of the total is 28

Total = $28 \div \frac{7}{30} = 120$

Silver balloons = $\frac{3}{5}$ of 120 = 72

Answer: 72

3 Hollow cylinder and density

A metal pipe is a hollow cylinder with outer radius 6 cm and inner radius 5 cm. (Calculator)
The density of the metal is 8.5 g/cm^3.
The mass of the pipe is 14.7 kg.
Work out the length of the pipe. Give your answer in centimetres correct to 3 significant figures.

Mass = 14700 g, so volume = mass ÷ density = 14700 ÷ 8.5 = 1729.4 cm^3
Cross-sectional area = $\pi \times 6^2 - \pi \times 5^2 = 34.558$ cm^2
Length = volume ÷ area = 1729.4 ÷ 34.558 = 50.0 cm (3 s.f.)
Answer: 50.0 cm

4 Local maximum and minimum values

$f(x) = 2x^3 - 54x - 4$
Find the y coordinate of the local maximum point and the y coordinate of the local minimum point.

$f'(x) = 6x^2 - 54 = 6(x - 3)(x + 3)$
$f'(x) = 0$ gives $x = -3$ or $x = 3$
$f''(x) = 12x$: $f''(-3) = -36 < 0$ (maximum) and $f''(3) = 36 > 0$ (minimum)
$f(-3) = 104$ and $f(3) = -112$
Answer: Local maximum y = 104, local minimum y = −112

5 Speed-time graphs

The graph shows the speed of a train between two stations.
(a) Work out the acceleration in the first 6 seconds.
(b) Work out the total distance.

(a) Acceleration = change in speed ÷ time = 12 ÷ 6 = 2 m/s^2
(b) Distance = area under the graph, split into two triangles and a rectangle
$\frac{1}{2} \times 6 \times 12 + 12 \times 15 + \frac{1}{2} \times 3 \times 12 = 36 + 180 + 18$
= 234
Answer: (a) 2 m/s^2 (b) 234 m

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6 Successive percentage change

The value of a car is decreased by 10%.
The new amount is then decreased by 10%.
Work out the overall percentage change.

Multipliers: 0.9 and 0.9
0.9 × 0.9 = 0.81
0.81 is a decrease of 19%
Answer: 19% decrease

7 Same gradient on two curves

Two curves have equations $y = 3x^2 - 6x + 5$ and $y = x^2 + 6x - 2$.
Find the value of x at which the curves have the same gradient.

First curve: $\frac{dy}{dx} = 6x - 6$

Second curve: $\frac{dy}{dx} = 2x + 6$

Equal gradients: $6x - 6 = 2x + 6$
$4x = 12$, so $x = 3$
Answer: x = 3

8 Tangent to a circle

A circle has equation $x^2 + y^2 = 169$.
The point P(−5, 12) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{12}{5}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{12}{5}) = \frac{5}{12}$

$y - 12 = \frac{5}{12}(x - (-5))$

Multiply out and rearrange: $5x - 12y = -169$
(b) On the x-axis $y = 0$, so $5x = -169$
$x = -\frac{169}{5}$, so T is $(-\frac{169}{5}, 0)$

Answer: (a) $5x - 12y = -169$ (b) T = $(-\frac{169}{5}, 0)$

9 Gradient where the curve crosses the x-axis

A curve has equation $y = (x + 1)(x - 3)(x - 4)$.
Work out the gradient of the curve at each point where it crosses the x-axis.

Expand: $y = x^3 - 6x^2 + 5x + 12$
$\frac{dy}{dx} = 3x^2 - 12x + 5$
The curve crosses the x-axis at $x = -1$, 3 and 4
Gradients: $x = -1$ gives 20, $x = 3$ gives −4, $x = 4$ gives 5
Answer: 20 at x = −1, −4 at x = 3, 5 at x = 4

10 Venn diagram: reading set notation

The Venn diagram shows information about 39 customers.
A: ordered tea B: ordered coffee
(a) Find n(A′).
(b) Find n(A ∪ B)′.

(a) n(A′) = 13 + 3 = 16
(b) n(A ∪ B)′ = 3
Answer: (a) 16 (b) 3