Day 123 **IGCSE 10-a-day: Model Solutions**

1 Projectile height

A ball is thrown upwards. Its height above the ground, h metres, after t seconds is given by

$h = -2t^2 + 16t$

(a) Work out the greatest height reached by the ball.

(b) Work out the velocity of the ball when t = 9. State whether it is moving up or down.

$\frac{dh}{dt} = -4t + 16 = 0$ gives $t = 4$

$\frac{d^2h}{dt^2} = -4 < 0$ so maximum. $h = 32$

When $t = 9$: $\frac{dh}{dt} = 16 - 4 \times 9 = -20$

Answer: (a) 32 m (b) −20 m/s, moving down

2 Venn diagram: reading set notation

The Venn diagram shows information about 24 families.

A: have a garden B: have a car

(a) Find $n(A \cup B)'$.

(b) Find $n(A \cup B)$.

(a) $n(A \cup B)' = 3$

(b) $n(A \cup B) = 2 + 6 + 13 = 21$

Answer: (a) 3 (b) 21

3 Gradient with fractional or negative powers

The equation of a curve is $y = -2x^{3/2} + 3x - 8\sqrt{x}$.

Work out the gradient of the curve at the point where x = 9.

$\frac{dy}{dx} = -3\sqrt{x} + 3 - 4x^{-1/2}$

When $x = 9$: gradient $= -\frac{22}{3}$

Answer: $-\frac{22}{3}$

4 Expanding three brackets

Expand and simplify $(2x + 3)(x + 2)(x - 1)$

Expand the first two brackets: $(x + 2)(x - 1) = x^2 + x - 2$

Multiply by $(2x + 3)$: $2x(x^2 + x - 2) + 3(x^2 + x - 2)$

$= 2x^3 + 5x^2 - x - 6$

Answer: $2x^3 + 5x^2 - x - 6$

5 Independent events and probability rules

A and B are two events.

P(A) = 0.8, P(B) = 0.4 and P(A or B) = 0.88

(a) Work out P(A and B).

(b) Are A and B independent? You must show your working.

(a) $P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$0.88 = 0.8 + 0.4 - P(A \cap B)$, so $P(A \cap B) = 0.32$

(b) $P(A) \times P(B) = 0.8 \times 0.4 = 0.32$

This equals $P(A \cap B)$, so A and B are independent

Answer: (a) 0.32 (b) Yes

6 Circle and line intersections

Solve the simultaneous equations

$x^2 + y^2 = 25$

$x + y = 1$

From the second equation, $y = -x + 1$

Substitute into the first equation: $x^2 + (-x + 1)^2 = 25$

Expand: $2x^2 - 2x - 24 = 0$

Divide by 2: $x^2 - x - 12 = 0$

Factorise: $(x + 3)(x - 4) = 0$, so $x = -3$ or $x = 4$

$y = -x + 1$ gives $y = 4$ and $y = -3$

Answer: $x = -3$, $y = 4$ and $x = 4$, $y = -3$

7 HCF and LCM

The HCF of two numbers is 8 and their LCM is 264.

The smaller number is 24.

Work out the larger number.

HCF × LCM = product of the two numbers

$8 \times 264 = 2112$

Larger number = $2112 \div 24 = 88$

Answer: 88

8 Loci and regions

ABCD is a rectangle with AB = 6 cm and AD = 10 cm.

A point P inside the rectangle is such that AP ≤ 6 cm and P is nearer to A than to B.

Work out the area of the region in which P can lie. Give your answer in terms of π and surds.

Nearer to A than to B: P is on the A side of the perpendicular bisector of AB, a distance 3 cm from A

AP ≤ 6 cm: P is inside the quarter circle of radius 6 cm centred at A

The bisector meets the arc where $\cos\theta = \frac{3}{6}$, so $\theta = 60°$ at A. The region is a right-angled triangle plus a sector of angle 30°

Triangle: base 3 cm, height $6\sin 60° = 3\sqrt{3}$ cm, area $= \frac{1}{2} \times 3 \times 3\sqrt{3} = \frac{9}{2}\sqrt{3}$

Sector of angle 30° : $\frac{30}{360} \times \pi \times 6^2 = 3\pi$

Area $= \frac{9}{2}\sqrt{3} + 3\pi$ cm^2

Answer: $\left(\frac{9}{2}\sqrt{3} + 3\pi\right)$ cm^2

9 Kinematics with roots

A particle moves in a straight line. After t seconds its displacement from O is s metres, where

$s = 24t^{\frac{3}{2}} + t^2$ $(t \geq 0)$

Find the velocity and the acceleration of the particle when t = 4.

$v = \frac{ds}{dt} = 2t + 36t^{\frac{1}{2}}$

$a = \frac{dv}{dt} = 2 + 18t^{-\frac{1}{2}}$

When t = 4: v = 80 and a = 11

Answer: v = 80 m/s, a = 11 m/s^2

10 Bounds of a compound expression

p = 12.5 and r = 8.9, each correct to 1 decimal place.
q = 2.99, correct to 2 decimal places.

Work out the lower bound of $\frac{p + q}{r}$. Give your answer correct to 3 significant figures. (Calculator)

For the lower bound use LB of the top and UB of the bottom
Top: LB p + LB q = 12.45 + 2.985 = 15.435
Bottom: UB r = 8.95
15.435 ÷ 8.95 = 1.72
Answer: 1.72