Day 122 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -4$, $q = -4$ and $r = -1$.

Substitute the values: $(-4 - (-4))^2 - (-4 + (-4))^2$
$= 0 - 64$
$= -64$
Answer: -64

2 Function values and composites

$f(x) = 5x - 7$ and $g(x) = 2x^2 - x - 4$.
Work out $ff(4) + gf(3)$.

$f(4) = 13$, so $ff(4) = f(13) = 58$
$f(3) = 8$, so $gf(3) = g(8) = 116$
$58 + 116 = 174$
Answer: 174

3 Tangents and circle theorems

PA and PB are tangents to a circle, centre O. C is a point on the circle. Angle ACB = 58°.
(a) Work out the size of angle AOB.
(b) Work out the size of angle APB.
The diagram is not drawn accurately.

(a) The angle at the centre is twice the angle at the circumference: $2 \times 58 = 116°$
(b) Angles OAP and OBP are 90° (a tangent meets a radius at 90°)
Angle APB = $360 - 90 - 90 - 116 = 64°$
Answer: (a) 116° (b) 64°

4 Increasing for x greater than k

The function $f(x) = x^2 + 8x + 6$ is increasing when $x > k$.
Find the value of k.

$f'(x) = 2x + 8$
The turning point is where $f'(x) = 0$: $2x + 8 = 0$
$x = -4$
$f'(x)$ is positive for $x > -4$, so $k = -4$
Answer: $k = -4$

5 Fractions of fractions

Uma spent $\frac{1}{7}$ of her money on a phone.

She then spent $\frac{1}{3}$ of the money that was left on a case.

A case cost £120.
How much money did Uma have to start with?

The second amount is $\frac{1}{3}$ of $\frac{6}{7}$, which is $\frac{2}{7}$ of the total

$\frac{2}{7}$ of the total is £120

Total = £120 ÷ $\frac{2}{7}$ = £420

Answer: £420

6 Where is the second derivative zero?

$y = -x^3 - 6x^2 + 9x + 4$

(a) Find $\frac{d^2y}{dx^2}$.

(b) Find the value of x for which $\frac{d^2y}{dx^2} = 0$, and work out the gradient of the curve at this value of x.

$\frac{dy}{dx} = -3x^2 - 12x + 9$

$\frac{d^2y}{dx^2} = -6x - 12$

$\frac{d^2y}{dx^2} = 0$ gives $-6x - 12 = 0$, so $x = -2$

$\frac{dy}{dx} = 21$ when $x = -2$

Answer: (a) $-6x - 12$ (b) $x = -2$, gradient 21

7 Pythagoras and quadratic equations

The lengths of the sides of a right-angled triangle are $(x - 1)$ cm, $(x + 1)$ cm and $(x + 3)$ cm.
The longest side is $(x + 3)$ cm.

(a) Show that $x^2 - 6x - 7 = 0$

(b) Hence work out the area of the triangle.

(a) Pythagoras: $(x - 1)^2 + (x + 1)^2 = (x + 3)^2$

Expand: $x^2 - 2x + 1 + x^2 + 2x + 1 = x^2 + 6x + 9$

Rearrange: $x^2 - 6x - 7 = 0$

(b) Factorise: $(x - 7)(x + 1) = 0$, so $x = 7$ (the other root, $x = -1$, does not give valid side lengths)

The shorter sides are 6 cm and 8 cm, so area $= \frac{1}{2} \times 6 \times 8 = 24$ cm^2

Answer: (b) 24 cm^2

8 Turning point after a transformation

The graph of $y = f(x)$ has a maximum point at (5, 3).
Write down the coordinates and the type of turning point on each graph.

(a) $y = f(-x)$

(b) $y = -f(x)$

(a) $y = f(-x)$: reflection in the y-axis: x-coordinate changes sign
$(-5, 3)$, a maximum

(b) $y = -f(x)$: reflection in the x-axis turns a maximum into a minimum
$(5, -3)$, a minimum

Answer: (a) $(-5, 3)$ maximum (b) $(5, -3)$ minimum

9 Wire cut in two

A piece of thin metal rod 140 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is three times its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{3(140 - 4x)^2}{64}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $140 - 4x = 8w$, so $w = \frac{140 - 4x}{8}$ and its area is $3w^2 = \frac{3(140 - 4x)^2}{64}$

$A = x^2 + \frac{3(140 - 4x)^2}{64} = \frac{7}{4}x^2 - \frac{105}{2}x + \frac{3675}{4}$

$\frac{dA}{dx} = \frac{7}{2}x - \frac{105}{2} = 0$, so $x = 15$. $\frac{d^2A}{dx^2} = \frac{7}{2} > 0$, so a minimum

Square: 225, rectangle: $30 \times 10 = 300$, so $A = 525$

Answer: x = 15, least area 525 cm^2

10 Venn diagrams with algebra

The Venn diagram shows how many of 37 students play the piano (P) or the guitar (G).

(a) Work out the value of x.

(b) A student is chosen at random from those who play the guitar. Work out the probability that this student also plays the piano.

(a) Total: $(x + 2) + x + (2x - 1) + 4 = 37$

$4x + 5 = 37$, so $x = 8$

(b) Students in the second circle: $x + 2x - 1 = 8 + 15 = 23$, of whom 8 are in both

Probability = $\frac{8}{23}$

Answer: (a) x = 8 (b) $\frac{8}{23}$