Day 121 **IGCSE 10-a-day: Model Solutions**

1 Substitution

Work out the value of $(p - q)^2 - (p + q)^2$
when $p = -1$, $q = -1$ and $r = 1$.

Substitute the values: $(-1 - (-1))^2 - (-1 + (-1))^2$
$= 0 - 4$
$= -4$
Answer: -4

2 Average speed

Maya drives 45 km at an average speed of 45 km/h.
The drive then continues for another 105 km at an average speed of 35 km/h.
Work out the average speed for the whole journey.

Time for first part = 45 ÷ 45 = 1 hour
Time for second part = 105 ÷ 35 = 3 hours
Total distance = 150 km, total time = 4 hours
Average speed = 150 ÷ 4 = 37.5 km/h
Answer: 37.5 km/h

3 Compound interest

Chen invests £3,500.
The account pays compound interest of 4% a year for the first 2 years,
then 2.5% a year for the next 2 years.
Work out the total amount after 4 years, to the nearest penny. (Calculator)

Multipliers: 1.04 for 2 years, then 1.025 for 2 years
$£3{,}500 \times 1.04^2 \times 1.025^2$
= £3,977.25
Answer: £3,977.25

4 Venn diagram: reading set notation

The Venn diagram shows information about 41 students.
A: study French B: study Spanish
(a) Find $n(A \cap B')$.
(b) Find $n(B')$.

(a) $n(A \cap B') = 15$
(b) $n(B') = 15 + 5 = 20$
Answer: (a) 15 (b) 20

5 Points with a given gradient (cubic)

The curve $y = x^3 - 3x^2 - x$ has gradient 8 at two points.
Find the coordinates of these points.

$\frac{dy}{dx} = 3x^2 - 6x - 1$

$3x^2 - 6x - 1 = 8$, so $3x^2 - 6x - 9 = 0$
Divide by 3: $x^2 - 2x - 3 = 0$
$(x + 1)(x - 3) = 0$, so $x = -1$ or $x = 3$
Substitute into y: $(-1, -3)$ and $(3, -3)$
Answer: $(-1, -3)$ and $(3, -3)$

6 Area of a triangle given three sides

Work out the area of triangle ABC.
Give your answer correct to 3 significant figures. (Calculator)

$\cos A = \frac{16^2 + 9^2 - 12^2}{2 \times 16 \times 9} = \frac{193}{288}$

$A = 47.922°$

Area = ½ × 16 × 9 × sin 47.922° = 53.4

Answer: 53.4 cm^2

7 Open box from a square sheet

Squares of side x cm are cut from the corners of a 48 cm square card sheet, which is folded into an open box of volume V cm^3.
(a) Show that $V = 4x^3 - 192x^2 + 2304x$.
(b) Find x for the maximum volume, and this volume.

Base is a square of side 48 − 2x, so $V = x(48 - 2x)^2 = x(2304 - 192x + 4x^2) = 4x^3 - 192x^2 + 2304x$

$\frac{dV}{dx} = 12x^2 - 384x + 2304 = 0$, so $x^2 - 32x + 192 = 0$

$12(x - 8)(x - 24) = 0$, so x = 8 or x = 24

x = 24 is rejected (24 is more than half of 48). $\frac{d^2V}{dx^2} = 24x - 384$, which is −192 when x = 8, so a maximum

V = 8192

Answer: x = 8, V = 8192 cm^3

8 Bearings with the sine and cosine rules

A plane leaves an airport P and travels 115 km on a bearing of 010° to Q. (Calculator)
It then travels 102 km on a bearing of 255° to R.
(a) Work out the distance PR.
(b) Work out the bearing of R from P.
Give distances to 1 decimal place and the bearing to the nearest degree.

(a) The bearing of P from Q is 010° + 180° = 190°

Angle PQR = 65° (the difference between the bearings 190° and 255°)

$PR^2 = 115^2 + 102^2 - 2 \times 115 \times 102 \times \cos 65° = 13714$, so PR = 117.1 km

(b) $\cos(QPR) = \frac{115^2 + PR^2 - 102^2}{2 \times 115 \times PR}$, so angle QPR = 52.1°

R is anticlockwise of the direction PQ, so the bearing is 010° − 52° = 318°

Answer: (a) 117.1 km (b) 318°

9 Three stationary points of a quartic

$y = x^4 - 18x^2 + 4$
Find the coordinates of the stationary points of the curve and determine the nature of each.

$\frac{dy}{dx} = 4x^3 - 36x = 4x(x - 3)(x + 3)$

$\frac{dy}{dx} = 0$ gives x = −3, x = 0 or x = 3

Points: (−3, −77), (0, 4), (3, −77)

$\frac{d^2y}{dx^2} = 12x^2 - 36$

x = −3: $\frac{d^2y}{dx^2} = 72$; x = 0: $\frac{d^2y}{dx^2} = -36$; x = 3: $\frac{d^2y}{dx^2} = 72$

So (−3, −77) is a minimum, (0, 4) is a maximum, (3, −77) is a minimum

Answer: (−3, −77) minimum; (0, 4) maximum; (3, −77) minimum

10 Tangent to a circle

A circle has equation $x^2 + y^2 = 289$.
The point P(−15, 8) lies on the circle.
(a) Find the equation of the tangent to the circle at P. Give your answer in the form ax + by = c, where a, b and c are integers.
(b) The tangent meets the x-axis at the point T. Write down the coordinates of T.

(a) The gradient of the radius OP is $-\frac{8}{15}$

The tangent is perpendicular to the radius, so its gradient is $-1 \div (-\frac{8}{15}) = \frac{15}{8}$

$y - 8 = \frac{15}{8}(x - (-15))$

Multiply out and rearrange: $15x - 8y = -289$
(b) On the x-axis y = 0, so $15x = -289$
$x = -\frac{289}{15}$, so T is $(-\frac{289}{15}, 0)$

Answer: (a) $15x - 8y = -289$ (b) T = $(-\frac{289}{15}, 0)$