Day 120 **IGCSE 10-a-day: Model Solutions**

1 Quadratic sequences

Here are the first five terms of a quadratic sequence.
5, 3, −3, −13, −27
Find an expression for the nth term.

First differences: −2, −6, −10, −14
Second differences: −4, −4, −4, so a = −4 ÷ 2 = −2
Subtract $-2n^2$ from each term: 7, 11, 15, 19, 23
The remaining linear sequence has nth term 4n + 3
nth term = $-2n^2 + 4n + 3$
Answer: $-2n^2 + 4n + 3$

2 Angle in a semicircle

AB is a diameter of a circle, centre O. C is a point on the circle. Angle BAC = 58°.
(a) Work out the size of angle ACB.
(b) Work out the size of angle COB.
The diagram is not drawn accurately.

(a) The angle in a semicircle is 90°, so angle ACB = 90°
(b) OA = OC (radii), so triangle OAC is isosceles and angle OCA = 58°
Angle COB is an exterior angle of triangle OAC: 58 + 58 = 116°
(or: the angle at the centre is twice the angle at the circumference)
Answer: (a) 90° (b) 116°

3 Multiplying and dividing surds

Work out $(2\sqrt{5})^2 - (4\sqrt{5})^2$

$(2\sqrt{5})^2 = 2^2 \times 5 = 20$
$(4\sqrt{5})^2 = 4^2 \times 5 = 80$
20 − 80 = −60
Answer: −60

4 No stationary points

A curve has equation $y = 2x^3 + 9x + 9$.
(a) Show that the curve has no stationary points.
(b) Find the least value of the gradient of the curve.

$\frac{dy}{dx} = 6x^2 + 9 = 6x^2 + 9$

A squared term is never negative, so $\frac{dy}{dx} \geq 9 > 0$ for all x

So $\frac{dy}{dx}$ is never zero, and there are no stationary points

The least gradient is 9 (when x = 0)
Answer: (b) 9

5 Exponential model y = ab^t

$V = ab^n$, where a and b are positive constants.
When n = 2, V = 2880. When n = 4, V = 4147.2.
Find the values of a and b, and the value of V when n = 6.

Divide: $b^2 = 4147.2 \div 2880 = 1.44$
b = 1.2
$a \times 1.2^2 = 2880$ so a = 2000
$V = 2000 \times 1.2^6 = 5971.968$
Answer: a = 2000, b = 1.2, V = 5971.968

6 Enlargement

Triangle P has vertices A(4, 10), B(2, 4) and C(8, 4).
Triangle Q has vertices A'(4, 1), B'(5, 4) and C'(2, 4).
Q is an enlargement of P.
Describe fully the single transformation that maps P onto Q.

x-step from A to B = −2, x-step from A' to B' = 1

Scale factor = $1 \div -2 = -\frac{1}{2}$

The centre C satisfies: $C + -\frac{1}{2}(A - C) = A'$, so $(1 - -\frac{1}{2})C = A' - -\frac{1}{2}A$

x: $\frac{3}{2} \times x = 4 - (-\frac{1}{2}) \times 4 = 6$, so x = 4

y: $\frac{3}{2} \times y = 1 - (-\frac{1}{2}) \times 10 = 6$, so y = 4

Answer: Enlargement, scale factor $-\frac{1}{2}$, centre (4, 4)

7 Combined and changing means

The mean of 9 numbers is 36.
One of the numbers, 45, is removed.
Work out the mean of the remaining 8 numbers.

Total of 9 numbers = 9 × 36 = 324
Total after removing 45 = 324 − 45 = 279
New mean = 279 ÷ 8 = 34.875
Answer: 34.875

8 Normal in the form ax + by + c = 0

The point A lies on the curve $y = -x^2 + 6x + 7$ where x = 2.
Find the equation of the normal at A. Give your answer in the form ax + by + c = 0 where a, b and c are integers.

y = 15 at x = 2 and gradient of curve = 2

Gradient of normal = $-\frac{1}{2}$

$y - 15 = -\frac{1}{2}(x - 2)$

$y = -\frac{1}{2}x + 16$

Multiply by 2: x + 2y − 32 = 0
Answer: x + 2y − 32 = 0

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to t.
When v = 4 and t = 2, P = 8.
Work out the value of P when v = 4 and t = 4.

$P = \frac{kv^2}{t}$

$8 = \frac{k \times 16}{2}$, so $k = 8 \times 2 \div 16 = 1$

$P = \frac{1 \times 16}{4} = 4$

Answer: P = 4

10 Increasing or decreasing at a point

$y = 2x^3 - 24x + 5$

At the point where $x = -3$, is the curve increasing or decreasing? You must show your working.

$\frac{dy}{dx} = 6x^2 - 24$

When $x = -3$: $\frac{dy}{dx} = 30$

30 is positive, so the curve is increasing

Answer: increasing (gradient 30)