Day 12 **IGCSE 10-a-day: Model Solutions**

1 Show that: consecutive terms of a quadratic sequence

The nth term of a sequence is $3n^2 - 3n$.
(a) Show that the difference between the (n + 1)th term and the nth term is 6n.
(b) Two consecutive terms of the sequence differ by 102. Find the value of n for the first of these two terms.

(a) (n + 1)th term = $3(n + 1)^2 - 3(n + 1) + 0 = 3n^2 + 3n$
Subtract the nth term: $3n^2 + 3n - (3n^2 - 3n) = 6n$
(b) 6n = 102 so 6n = 102
n = 17
Answer: (b) n = 17

2 Angles with algebra

In triangle PQR, side QR is extended to S.
Angle QPR = $(2x - 20)°$
Angle PQR = $(2x + 25)°$
Angle PRS = $(3x + 32)°$
(a) Work out the value of x.
(b) Work out the size of angle PRQ.

(a) The exterior angle of a triangle equals the sum of the two opposite interior angles
3x + 32 = 4x + 5
x = 27
(b) Angle PRS = 113° so angle PRQ = 180° − 113° = 67°
Answer: (a) x = 27 (b) 67°

3 Arithmetic sequences

The 2nd term of an arithmetic sequence is 7.
The 6th term of the sequence is 19.
(a) Find an expression for the nth term of the sequence.
(b) Is 82 a term of the sequence? You must show your working.

(a) Common difference = (19 − 7) ÷ (6 − 2) = 3
nth term = 3n + c. For n = 2: 3 × 2 + c = 7, so c = 1
nth term = 3n + 1
(b) Solve 3n + 1 = 82: 3n = 81, so n = 27
n is a positive whole number, so 82 is the 27th term
Answer: (a) 3n + 1 (b) Yes, it is the 27th term

4 Product of brackets: tangent gradient

y = x(x + 4)(2x + 3)
Work out the gradient of the curve at the point where x = 1.

Expand: $y = 2x^3 + 11x^2 + 12x$
$\frac{dy}{dx} = 6x^2 + 22x + 12$
x = 1: $\frac{dy}{dx} = 40$
Answer: 40

5 Line of best fit

The scatter graph shows temperature (x) and ice creams sold (y).
The line of best fit passes through (4, 28) and (6, 36).
(a) Find its equation in the form y = mx + c.
(b) Explain what its gradient tells you.

(a) Gradient m = $\frac{36 - 28}{6 - 4} = \frac{8}{2} = 4$
y = 4x + c: using (4, 28), 28 = 4 × 4 + c, so c = 12
y = 4x + 12
(b) For each extra degree of temperature, the number of ice creams sold goes up by 4
Answer: (a) y = 4x + 12 (b) For each extra degree of temperature, the number of ice creams sold goes up by 4

6 Show that dy/dx factorises

$y = (x - 3)^2(x + 1)$

Show that $\frac{dy}{dx} = (x - 3)(3x - 1)$.

Expand: $y = x^3 - 5x^2 + 3x + 9$

$\frac{dy}{dx} = 3x^2 - 10x + 3$

Factorise: $3x^2 - 10x + 3 = (x - 3)(3x - 1)$

Answer: $\frac{dy}{dx} = (x - 3)(3x - 1)$

7 Inverse functions

$f(x) = \frac{4}{x + 4} + 4$

(a) Find $f^{-1}(x)$.

(b) Find the value of $f^{-1}(8)$.

(a) Let $y = \frac{4}{x + 4} + 4$, so $y - 4 = \frac{4}{x + 4}$

Make x the subject: $x + 4 = \frac{4}{y - 4}$

$x = \frac{4}{y - 4} - 4$

$f^{-1}(x) = \frac{4}{x - 4} - 4$

(b) $f^{-1}(8) = \frac{4}{8 - 4} - 4 = -3$

Answer: (a) $f^{-1}(x) = \frac{4}{x - 4} - 4$ (b) -3

8 Vertical distance between stationary points

The curve $y = 2x^3 + 3x^2 - 12x + 6$ has two stationary points, A and B.

Work out the vertical distance between A and B.

$\frac{dy}{dx} = 6x^2 + 6x - 12 = 6(x + 2)(x - 1)$

$x = -2$ and $x = 1$

At $x = -2$, $y = 26$; at $x = 1$, $y = -1$

Difference $= 26 - (-1) = 27$

Answer: 27

9 Three sets from information

69 students were asked which activities they do.

A: study Art B: study Music C: study Drama

$n(A) = 43$, $n(B) = 31$, $n(C) = 46$, $n(A \cap B) = 19$, $n(A \cap C) = 27$, $n(B \cap C) = 19$, $n(A \cap B \cap C) = 13$.

(a) Work out how many are in none of A, B and C.

(b) Work out how many are in exactly one of A, B and C.

$n(A \cup B \cup C) = 43 + 31 + 46 - 19 - 27 - 19 + 13 = 68$

(a) Number in none of the sets $= 69 - 68 = 1$

(b) Only $A = 43 - 19 - 27 + 13 = 10$

Only B = 6, only C = 13, so exactly one $= 10 + 6 + 13 = 29$

Answer: (a) 1 (b) 29

10 Nested surds

Show that $\sqrt{49 + 20\sqrt{6}} = 5 + 2\sqrt{6}$

Square the right-hand side: $(5 + 2\sqrt{6})^2 = 25 + 20\sqrt{6} + 24$

$= 49 + 20\sqrt{6}$

So $\sqrt{49 + 20\sqrt{6}} = 5 + 2\sqrt{6}$

Answer: $\boldsymbol{\sqrt{49 + 20\sqrt{6}} = 5 + 2\sqrt{6}}$