Day 119 **IGCSE 10-a-day: Model Solutions**

1 Solve f(x) = g(x)

f(x) = $2x^2 + 4x + 3$ and g(x) = $14x - 9$.
Solve f(x) = g(x).

$2x^2 + 4x + 3 = 14x - 9$
$2x^2 - 10x + 12 = 0$
$2(x - 3)(x - 2) = 0$
$x = 2$ or $x = 3$
Answer: x = 2 or x = 3

2 Equations with fractions

Solve $\frac{x-3}{8} - \frac{x-7}{2} = 5$

Multiply every term by 8: $(x - 3) - 4(x - 7) = 40$
Expand: $-3x + 25 = 40$
$-3x = 15$
$x = -5$
Answer: x = −5

3 Rate of change of area or volume

The volume, V cm^3, of a cube of edge x cm is $V = x^3$.

(a) Find $\frac{dV}{dx}$ when x = 2.

(b) Find the value of x when $\frac{dV}{dx} = 243$.

$\frac{dV}{dx} = 3x^2$

When x = 2: $\frac{dV}{dx} = 12$

$3x^2 = 243$, so $3x^2 = 243$ and x = 9 (lengths are positive)
Answer: (a) 12 (b) x = 9

4 Inverse proportion in context

9 painters can paint a hall in 24 days.
After 3 days, 6 of the painters leave. The rest carry on working at the same rate.
How many more days does it take to finish?

Total work = 9 × 24 = 216 painter-days
Work done = 9 × 3 = 27, so work left = 189
Workers left = 9 − 6 = 3
Time = 189 ÷ 3 = 63 days
Answer: 63 days

5 Equation of a normal

A curve has equation $y = 3x^3 - 2x + 4$.
Find the equation of the normal to the curve at the point where x = 0.

When x = 0, y = 4 and $\frac{dy}{dx} = -2$

Gradient of the normal = $-1 \div (-2) = \frac{1}{2}$

$y - 4 = \frac{1}{2}x$

$y = \frac{1}{2}x + 4$

Answer: $y = \frac{1}{2}x + 4$

6 Estimating the median from grouped data

The table gives information about 92 lengths.
Work out an estimate for the median.
Assume the values in each class are evenly spread.

Cumulative frequencies: 16, 21, 44, 69, 92
The median is the value at position 92 ÷ 2 = 46
It is in the class 30 to 40, with 44 values before it, so we need 2 more of the 25 values in this class

Median $\approx 30 + \frac{2}{25} \times 10 = 30.8$

Answer: 30.8

7 Angles of elevation

A point A is on level ground in line with the base of a tower. (Calculator)
The angle of elevation of the top of the tower from A is 44°.
B is a point on the same line, 100 m further from the tower than A.
The angle of elevation of the top of the tower from B is 27°.
Work out the height of the tower. Give your answer to 1 decimal place.

Let the height be h and the distance from A to the base be x. $\tan 44° = \frac{h}{x}$, so $x = h \div \tan 44°$

From B: $\tan 27° = \frac{h}{x + 100}$, so $x + 100 = h \div \tan 27°$

Subtract: $100 = h \div \tan 27° - h \div \tan 44° = h(1.9626 - 1.0355)$
h = 100 ÷ 0.92708 = 107.9 m (1 d.p.)
Answer: 107.9 m

8 Bounds: finding an unknown length

A rectangle has area 54 cm^2, correct to 2 significant figures.
Its length is 9.199999999999999289457264239899814128875732421875 cm, correct to 2 significant figures.
Work out the upper bound of the width of the rectangle. Give your answer correct to 3 significant figures. (Calculator)

Width = area ÷ length
Area: $53.5 \leq A < 54.5$. Length: $9.149999999999999289457264239899814128875732421875 \leq l < 9.249999999999999289457264239899814128875732421875$
UB width = UB area ÷ LB length = 54.5 ÷ 9.149999999999999289457264239899814128875732421875 = 5.96
Answer: 5.96 cm

9 Curves that touch

The curves $y = x^2 - 5x + 2$ and $y = -2x^2 + x - 1$ touch at one point.
Find the coordinates of this point and the equation of the common tangent.

Equate the two expressions: $3x^2 - 6x + 3 = 0$
That is $3(x - 1)^2 = 0$ with a repeated root x = 1
y = −2, so the point is (1, −2)
$\frac{dy}{dx} = 2x - 5$ on the first curve: gradient −3
Tangent: y = −3x + 1
Answer: (1, −2), y = −3x + 1

10 Growth and decay

A van is worth £10,000 when new.
Its value decreases by 12% each year.
After how many whole years will it first be worth less than £2,500? (Calculator)

Multiplier each year = 0.88

After 10 years: £10,000 × 0.88^{10} = £2,785.01

After 11 years: £10,000 × 0.88^{11} = £2,450.81

So the first time it is less than £2,500 is after 11 years

Answer: 11 years