Day 118 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $3x^2 + 4x - 7$
(a) Find f′(x).
(b) Work out the value of f′(−2).

(a) $3x^2 \rightarrow 6x$, $4x \rightarrow 4$
f′(x) = 6x + 4
(b) f′(−2) = −8
Answer: (a) f′(x) = 6x + 4 (b) −8

2 Error intervals

The height of a tower is 990 m, correct to 2 significant figures.
Write down the error interval for the height of a tower, using h.

The value has been rounded to the nearest 10
Half of this is 5, so go 5 below and 5 above
990 − 5 = 985 and 990 + 5 = 995
985 ≤ h < 995
Answer: 985 ≤ h < 995

3 Column vector arithmetic

$\mathbf{p} = \begin{pmatrix} -3 \\ -1 \end{pmatrix}$, $\mathbf{q} = \begin{pmatrix} -5 \\ 4 \end{pmatrix}$ and $\mathbf{r} = \begin{pmatrix} 1 \\ -6 \end{pmatrix}$.
Work out $\mathbf{p} + 2\mathbf{q} - \mathbf{r}$.

$2\mathbf{q} = \begin{pmatrix} -10 \\ 8 \end{pmatrix}$

$\mathbf{p} + 2\mathbf{q} - \mathbf{r} = \begin{pmatrix} -3 \\ -1 \end{pmatrix} + \begin{pmatrix} -10 \\ 8 \end{pmatrix} - \begin{pmatrix} 1 \\ -6 \end{pmatrix} = \begin{pmatrix} -14 \\ 13 \end{pmatrix}$

Answer: $\begin{pmatrix} \mathbf{-14} \\ \mathbf{13} \end{pmatrix}$

4 Cyclic quadrilaterals

A, B, C and D are points on a circle. Angle DAB = (4x + 20)°, angle BCD = (2x − 14)° and angle ABC = (2x)°.
(a) Work out the value of x.
(b) Work out the size of angle ADC.
The diagram is not drawn accurately.

(a) Opposite angles of a cyclic quadrilateral add up to 180°
(4x + 20) + (2x − 14) = 180
6x + 6 = 180, so 6x = 174
x = 29
(b) Angle ABC = 2 × 29 + 0 = 58°
Angle ADC = 180 − 58 = 122°
Answer: (a) x = 29 (b) 122°

5 Tangent to a curve with negative or fractional powers

A curve has equation $y = -2x^{3/2} + 4\sqrt{x}$.
Find the equation of the tangent to the curve at the point where x = 9.

$\frac{dy}{dx} = -3\sqrt{x} + 2x^{-1/2}$

At x = 9: y = −42 and gradient = $-\frac{25}{3}$

$y + 42 = -\frac{25}{3}(x - 9)$

$y = -\frac{25}{3}x + 33$

Answer: $\mathbf{y = -\frac{25}{3}x + 33}$

6 Sine rule

The diagram shows triangle ABC. (Calculator)
Work out the length of AC. Give your answer correct to 1 decimal place.
The diagram is not drawn accurately.

Use the sine rule: $\frac{AC}{\sin B} = \frac{BC}{\sin A}$

$\frac{x}{\sin 45°} = \frac{15}{\sin 79°}$

$x = \frac{15 \times \sin 45°}{\sin 79°} = 10.805$

x = 10.8 cm (1 d.p.)

Answer: AC = 10.8 cm

7 Points with a given gradient (cubic)

The curve $y = 2x^3 + 3x^2 - 30x + 2$ has gradient 6 at two points.
Find the coordinates of these points.

$\frac{dy}{dx} = 6x^2 + 6x - 30$

$6x^2 + 6x - 30 = 6$, so $6x^2 + 6x - 36 = 0$

Divide by 6: $x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$, so $x = -3$ or $x = 2$

Substitute into y: (−3, 65) and (2, −30)

Answer: (−3, 65) and (2, −30)

8 Compound interest: find the rate

£8,000 is invested for 3 years at r% per year compound interest.
At the end of 3 years the investment is worth £8,489.66.
Work out the value of r. (Calculator)

$8000 \times (1 + \frac{r}{100})^3 = £8{,}489.66$

$(1 + \frac{r}{100})^3 = 8489.66 \div 8000 = 1.0612075$

$1 + \frac{r}{100} = 1.02$

r = 2

Answer: r = 2

9 Finding the rate of interest

Rhys invested £16,500 in a savings account for 2 years.
Compound interest was paid at a fixed rate per year.
At the end of the 2 years the account held £18,191.25.
Work out the annual rate of interest. (Calculator)

Let the yearly multiplier be m, so $16500 \times m^2 = 18191.25$

$m^2 = 18191.25 \div 16500 = 1.1025$

$m = \sqrt{1.1025} = 1.05$

Rate = (1.05 − 1) × 100 = 5%

Answer: 5% per year

10 Tangency conditions

The line $y = kx - 5$ is a tangent to the curve $y = x^2 - 2$.
Find the possible values of k.
Give any surds in their simplest form.

Equate: $x^2 - 2 = kx - 5$

Rearrange: $x^2 - kx + 3 = 0$

One point of contact, so $b^2 - 4ac = 0$: $k^2 - 4 \times 3 = 0$

$k^2 = 12$

$k = \pm\sqrt{12} = \pm 2\sqrt{3}$

Answer: $k = \pm 2\sqrt{3}$