Day 117 **IGCSE 10-a-day: Model Solutions**

1 Stratified sampling

The table shows the number of members in each of four groups.
(a) A stratified sample of 28 is taken. Work out how many come from Group B.
(b) A different stratified sample has 33 from Group B. Work out the size of this sample.

Total = 450 + 550 + 200 + 200 = 1400

(a) Group B: $\frac{550}{1400} \times 28 = 11$

(b) Fraction sampled = $33 \div 550 = \frac{3}{50}$, so sample size = $\frac{3}{50} \times 1400 = 84$

Answer: (a) 11 (b) 84

2 f′(x) notation

$f(x) = 4x^3 + x^2 - 3x$
(a) Find f′(x).
(b) Work out the value of f′(3).

(a) $4x^3 \to 12x^2$, $x^2 \to 2x$, $-3x \to -3$
$f'(x) = 12x^2 + 2x - 3$
(b) f′(3) = 111

Answer: (a) $f'(x) = 12x^2 + 2x - 3$ (b) 111

3 Standard form problems

A library has 2.5×10^4 books.
Each book has 4×10^5 words.
Work out the total number of words in all the books.
Give your answer in standard form.

Total = $(2.5 \times 10^4) \times (4 \times 10^5)$
$2.5 \times 4 = 10$ and $10^4 \times 10^5 = 10^9$
$= 1 \times 10^{10}$

Answer: 1×10^{10} words

4 Similar shapes

Two shapes are mathematically similar.
The smaller has height 40 cm and area 20 cm^2.
The larger has area 7.2 cm^2.
Work out the height of the larger shape.

Ratio of areas = 7.2 : 20 = 9 : 25
Ratio of lengths = 3 : 5
Height = $40 \times \frac{3}{5} = 24$ cm

Answer: 24 cm

5 Triangle shapes from position vectors

A = (2, 1), B = (−3, −3) and C = (7, −3).
Show that triangle ABC is isosceles.

$\mathbf{AB} = \begin{pmatrix} -5 \\ -4 \end{pmatrix}$ and $\mathbf{AC} = \begin{pmatrix} 5 \\ -4 \end{pmatrix}$

$|AB|^2 = 25 + 16 = 41$ and $|AC|^2 = 25 + 16 = 41$
$|AB| = |AC| = \sqrt{41}$, so the triangle is isosceles
Answer: $|\mathbf{AB}| = |\mathbf{AC}|$

6 Simplifying surds

Write $\sqrt{24} - \sqrt{54} - \sqrt{6} + \sqrt{150}$ in the form $a\sqrt{6}$
where a is an integer.

$\sqrt{24} = 2\sqrt{6}$; $\sqrt{54} = 3\sqrt{6}$; $\sqrt{150} = 5\sqrt{6}$

$(2 - 3 - 1 + 5)\sqrt{6} = 3\sqrt{6}$

Answer: $3\sqrt{6}$

7 Where a line meets the axes

The line L has equation $20x + 21y = -420$.
L crosses the x-axis at the point A and the y-axis at the point B.
(a) Find the coordinates of A and B.
(b) Work out the length of AB.

(a) At A, $y = 0$: $20x = -420$, so $x = -21$. A is $(-21, 0)$
At B, $x = 0$: $21y = -420$, so $y = -20$. B is $(0, -20)$

(b) $AB^2 = 21^2 + 20^2 = 841$

$AB = \sqrt{841} = 29$

Answer: (a) A(−21, 0), B(0, −20) (b) 29

8 Show particle never at rest

A particle moves in a straight line. After t seconds its displacement from a fixed point is s metres, where
$s = 2t^3 - 24t^2 + 98t - 4$
(a) Show that the particle is never at rest.
(b) Find the least speed of the particle and the time it occurs.

$v = \frac{ds}{dt} = 6t^2 - 48t + 98 = 6(t - 4)^2 + 2$

A square is never negative, so $v \geq 2 > 0$ and P is never at rest
Least value of v occurs when $t = 4$, giving 2

Answer: (b) 2 m/s when t = 4 s

9 Bearings with sine and cosine rules

A ship sails 8 km from P to Q on a bearing of 064°.
It then sails 13 km from Q to R on a bearing of 186°.
(a) Work out PR (3 s.f.). (b) Work out the bearing of R from P (nearest degree). (Calculator)

Angle PQR = 064° + 180° − 186° = 58°

$PR^2 = 8^2 + 13^2 - 2 \times 8 \times 13 \times \cos 58°$ so PR = 11.1 km

$\cos QPR = \frac{8^2 + 11.080^2 - 13^2}{2 \times 8 \times 11.080}$, angle QPR = 84.25°

Bearing of R from P = 064° + 84.25° = 148°

Answer: (a) 11.1 km (b) 148°

10 Cylinder of fixed surface area

A closed cylinder (a tin) has radius r cm, height h cm and total surface area 96π cm^2.
(a) Show that its volume, V cm^3, is $V = 48\pi r - \pi r^3$.
(b) Find the maximum value of V in terms of π, and show that $h = 2r$ when V is a maximum.

$2\pi r^2 + 2\pi rh = 96\pi$, so $h = \frac{48 - r^2}{r}$

$V = \pi r^2 h = \pi r(48 - r^2) = 48\pi r - \pi r^3$

$\frac{dV}{dr} = 48\pi - 3\pi r^2 = 0$, so $r^2 = 16$ and $r = 4$

$\frac{d^2V}{dr^2} = -6\pi r < 0$, so a maximum. $h = \frac{48 - 16}{4} = 8 = 2r$

$V = \pi \times 4^2 \times 8 = 128\pi$

Answer: 128π cm^3