Day 116 **IGCSE 10-a-day: Model Solutions**

1 Angles in polygons

The interior angles of a pentagon are
$x°$, $(2x + 30)°$, $(2x - 25)°$, $(x + 30)°$, $(2x + 25)°$
(a) Work out the value of x.
(b) Work out the size of the largest angle.

(a) Sum of interior angles = $(5 - 2) \times 180 = 540°$
$8x + 60 = 540$, so $8x = 480$
$x = 60$
(b) The angles are $60°$, $150°$, $95°$, $90°$, $145°$
The largest is $150°$
Answer: (a) x = 60 (b) 150°

2 Particle at rest

A particle P moves in a straight line. After t seconds, its displacement from a fixed point O is s metres, where
$s = t^3 - 12t^2 + 45t + 3$ $(t \geq 0)$
Find the acceleration of P at each instant when it is at rest.

$v = \frac{ds}{dt} = 3t^2 - 24t + 45$

P is at rest when $v = 0$: $3t^2 - 24t + 45 = 0$
$3(t - 3)(t - 5) = 0$, so $t = 3$ or $t = 5$

$a = \frac{d^2s}{dt^2} = 6t - 24$

$t = 3$: $a = -6$. $t = 5$: $a = 6$
Answer: -6 m/s^2 when t = 3 and 6 m/s^2 when t = 5

3 Matrix times vector and order

$A = \begin{pmatrix} -4 & -1 \\ -1 & 3 \end{pmatrix}$ and $B = \begin{pmatrix} 6 \\ -5 \end{pmatrix}$.
(a) Write down the order of matrix A.
(b) Work out AB.

(a) 2 rows and 2 columns: 2×2
(b) $AB = \begin{pmatrix} (-4) \times 6 + (-1) \times (-5) \\ (-1) \times 6 + 3 \times (-5) \end{pmatrix}$
$AB = \begin{pmatrix} -19 \\ -21 \end{pmatrix}$
Answer: (a) 2×2 (b) $\begin{pmatrix} \mathbf{-19} \\ \mathbf{-21} \end{pmatrix}$

4 Compound interest

Amir puts £3,000 in a savings account.
Compound interest is paid at 1% in year 1, 2% in year 2 and 2% in year 3.
Work out the amount in the account at the end of year 3.
Give your answer to the nearest penny. (Calculator)

Multipliers: 1.01, 1.02, 1.02
£3,000 × 1.01 × 1.02 × 1.02
= £3,152.41
Answer: £3,152.41

5 Tangent at a turning point

The graph of $y = -2x^2 + 8x - 6$ has a turning point at P.
Find the equation of the tangent to the curve at P.

$\frac{dy}{dx} = -4x + 8$

At P, $\frac{dy}{dx} = 0$, so $-4x + 8 = 0$ and $x = 2$

$y = 2$ at P
The gradient is 0, so the tangent is horizontal: $y = 2$
Answer: $y = 2$

6 Average speed

A train travels the first half of a journey at 60 km/h.
It travels the second half of the same journey at 90 km/h.
Work out the average speed for the whole journey.

Let each half of the journey be 180 km (a common multiple of 60 and 90)
Time for first half = 180 ÷ 60 = 3 hours
Time for second half = 180 ÷ 90 = 2 hours
Average speed = total distance ÷ total time = 360 ÷ 5 = 72 km/h
Answer: 72 km/h

7 Changing the subject

Make r the subject of $P = 5\sqrt{r - 2} - 2$

Add 2: $P + 2 = 5\sqrt{r - 2}$

Divide by 5: $\frac{P + 2}{5} = \sqrt{r - 2}$

Square both sides: $\frac{(P + 2)^2}{25} = r - 2$

$r = \frac{(P + 2)^2}{25} + 2$

Answer: $r = \frac{(P + 2)^2}{25} + 2$

8 Maximum profit

A company makes and sells x thousand items. All amounts are in thousands of pounds.
The revenue is $R = x(15 - 2x)$ and the cost is $C = 4 + 7x$.
(a) Show that the profit $P = R - C$ is $P = -2x^2 + 8x - 4$.
(b) Find the number of items that must be sold for the profit to be a maximum, and this maximum profit.

$P = 15x - 2x^2 - (4 + 7x) = -2x^2 + 8x - 4$

$\frac{dP}{dx} = -4x + 8 = 0$ gives $x = 2$

$\frac{d^2P}{dx^2} = -4 < 0$, so a maximum

$P = 4$

Answer: 2000 items, maximum profit £4000

9 Fibonacci-type sequences

A sequence is defined by $u_{n+2} = u_{n+1} + u_n$.

$u_1 = 3$ and $u_2 = x$.

(a) Show that $u_5 = 3x + 6$.

(b) Given that $u_5 = 30$, find x and hence work out u_8.

(a) $u_3 = 3 + x$, $u_4 = u_3 + u_2 = 2x + 3$, $u_5 = u_4 + u_3 = 3x + 6$

(b) $3x + 6 = 30$ so $3x = 24$ and $x = 8$

Terms: 3, 8, 11, 19, 30, 49, 79, 128

$u_8 = 128$

Answer: $x = 8$, $u_8 = 128$

10 Percentages with an unknown rate

A population is increased by x% in one year and then increased by a further x% the next year.

Over the two years the population increases by 69%.

Work out the value of x.

Yearly multiplier = $1 + \frac{x}{100}$

Two years: $(1 + \frac{x}{100})^2 = 1.69$

Take the square root: $1 + \frac{x}{100} = 1.3$

$\frac{x}{100} = 0.3$, so $x = 30$

Answer: $x = 30$