Day 115 **IGCSE 10-a-day: Model Solutions**

1 Nested surds

Show that $\sqrt{19 + 8\sqrt{3}} = 4 + \sqrt{3}$

Square the right-hand side: $(4 + \sqrt{3})^2 = 16 + 8\sqrt{3} + 3$

$= 19 + 8\sqrt{3}$

So $\sqrt{19 + 8\sqrt{3}} = 4 + \sqrt{3}$

Answer: $\sqrt{19 + 8\sqrt{3}} = 4 + \sqrt{3}$

2 Error intervals

The number m is 45.6 correct to 3 significant figures.
Write down the error interval for m.

The last significant figure is in the 0.1 place, so the half-unit is 0.05

Lower bound = 45.6 − 0.05 = 45.55; upper bound = 45.6 + 0.05 = 45.65

Answer: $45.55 \leq m < 45.65$

3 Gradient from a tangent

The diagram shows part of the curve $y = 2x^2 - 3x$. The line shown is the tangent to the curve at the point P. (Not drawn accurately.)
(a) Use the two marked points on the tangent to work out its gradient.
(b) Use differentiation to find the x coordinate of P.

Gradient = $\frac{1 - (-3)}{3 - (-1)} = 1$

$\frac{dy}{dx} = 4x - 3$

$4x - 3 = 1$, so $x = 1$

Answer: (a) 1 (b) $x = 1$

4 Simplifying algebraic fractions

Simplify fully $\frac{4x^2 - 64}{x^2 - 5x + 4}$

Factorise the top: $4(x - 4)(x + 4)$

Factorise the bottom: $(x - 4)(x - 1)$

Cancel the common factor $(x - 4)$: $\frac{4(x + 4)}{x - 1}$

Answer: $\frac{4(x + 4)}{x - 1}$

5 Points with a given gradient (cubic)

The curve $y = 2x^3 + 12x^2 + 6x - 5$ has gradient -12 at two points.
Find the x coordinates of these points.

$\frac{dy}{dx} = 6x^2 + 24x + 6$

$6x^2 + 24x + 6 = -12$, so $6x^2 + 24x + 18 = 0$

Divide by 6: $x^2 + 4x + 3 = 0$

$(x + 3)(x + 1) = 0$, so $x = -3$ or $x = -1$

Answer: $x = -3$ **and** $x = -1$

6 Exponential growth and decay

There are 200 insects in a sample. The number of insects is multiplied by 3 every day.
After how many days will there first be more than 100,000 insects?

Number after n days = 200×3^n

n = 5: 48,600

n = 6: 145,800

145,800 is the first value above 100,000

Answer: 6 days

7 Find constants from a turning point

The curve $y = x^3 + ax^2 + bx + 4$ has stationary points at $x = -3$ and $x = -1$.
Find the values of a and b.

$\frac{dy}{dx} = 3x^2 + 2ax + b$

$x = -3$: $27 + -6a + b = 0$. $x = -1$: $3 + -2a + b = 0$

Subtract the equations: $24 + -4a = 0$, so $a = 6$

Then $b = 9$

Answer: a = 6, b = 9

8 Recasting solids

A solid metal sphere of radius 6 cm is melted down and recast into a solid cylinder of radius 8 cm.
Work out the height of the cylinder.

The volume of a sphere of radius r is $\frac{4}{3}\pi r^3$.

Volume of sphere = $\frac{4}{3} \times \pi \times 6^3 = 288\pi$

Volume of cylinder = $\pi \times 8^2 \times h = 64\pi h$

$64h = 288$

$h = \frac{9}{2}$ cm

Answer: $\frac{9}{2}$ cm

9 Collinear points with vectors

$\mathbf{OA} = 3\mathbf{b}$, $\mathbf{OB} = -\mathbf{a} + 7\mathbf{b}$ and $\mathbf{OC} = -2\mathbf{a} + 11\mathbf{b}$.
Show that A, B and C lie on a straight line.

$\mathbf{AB} = \mathbf{OB} - \mathbf{OA} = -\mathbf{a} + 4\mathbf{b}$

$\mathbf{AC} = \mathbf{OC} - \mathbf{OA} = -2\mathbf{a} + 8\mathbf{b}$

$\mathbf{AC} = 2\mathbf{AB}$

AC is a multiple of AB and they share the point A, so A, B and C are collinear

Answer: AC = 2AB, so A, B, C are collinear

10 Expanding three brackets

Expand and simplify $(2x - 3)(x - 5)(x + 5)$

Expand the first two brackets: $(x - 5)(x + 5) = x^2 - 25$

Multiply by $(2x - 3)$: $2x(x^2 - 25) - 3(x^2 - 25)$

$= 2x^3 - 3x^2 - 50x + 75$

Answer: $2x^3 - 3x^2 - 50x + 75$