Day 114 **IGCSE 10-a-day: Model Solutions**

1 Inverse variation

s is inversely proportional to the cube of t.
When t = 10, s = 0.45.
(a) Find a formula for s in terms of t.
(b) Find s when t = 4.

$s = \frac{k}{t^3}$

$0.45 = \frac{k}{10^3}$ so $k = 0.45 \times 1000 = 450$

$s = \frac{450}{t^3}$

$s = \frac{450}{64} = 7.03125$

Answer: (a) $s = \frac{450}{t^3}$ (b) s = 7.03125

2 Coordinates and gradient at the y-intercept

$y = -3x^4 - x^3 + 3x^2 - 3$
The curve crosses the y-axis at the point A.
Work out the coordinates of A and the gradient of the curve at A.

When x = 0, y = −3, so A is (0, −3)

$\frac{dy}{dx} = -12x^3 - 3x^2 + 6x$

When x = 0, $\frac{dy}{dx} = 0$

Answer: A = (0, −3), gradient 0

3 Probabilities with an unknown

A biased spinner lands on one of four outcomes with these probabilities.
red: 2x; blue: 3x; green: 2x + 0.05; yellow: 0.6
(a) Work out the value of x.
(b) The biased spinner is used 60 times. Work out an estimate for the number of times it lands on red.

(a) Probabilities add up to 1: 2x + 3x + 2x + 0.05 + 0.6 = 1
7x + 0.65 = 1, so 7x = 0.35
x = 0.05
(b) P(red) = 2 × 0.05 = 0.1, so 60 × 0.1 = 6
Answer: (a) x = 0.05 (b) 6

4 Laws of indices with algebra

Simplify $(16x^2y^6)^{\frac{1}{2}}$

Raise each part to the power $\frac{1}{2}$

$16^{\frac{1}{2}} = \sqrt{16} = 4$
$(x^2)^{\frac{1}{2}} = x^1$ and $(y^6)^{\frac{1}{2}} = y^3$
$= 4xy^3$
Answer: $4xy^3$

5 Probability with sets

n(ξ) = 100, n(A) = 36, n(B) = 35 and n(A ∩ B) = 8.
An element of ξ is chosen at random.
(a) Find the probability that it is in A ∪ B.
(b) Given that it is in B, find the probability that it is also in A.

n(A ∪ B) = 36 + 35 − 8 = 63

(a) $P(A \cup B) = \frac{63}{100}$

(b) Only the 35 elements in B can be chosen, and 8 of these are in A

$P = \frac{8}{35}$

Answer: (a) $\frac{63}{100}$ (b) $\frac{8}{35}$

6 Minimum surface area of a tank

A tank is a cuboid with a square base of side x metres and height h metres, and has an open top. Its volume is 500 m^3.
(a) Show that the total outside surface area, A m^2, is $A = x^2 + \frac{2000}{x}$.
(b) Find the minimum possible value of A, showing that it is a minimum.

$V = x^2h = 500$, so $h = \frac{500}{x^2}$

$A = x^2 + 4xh = x^2 + 4x \times \frac{500}{x^2} = x^2 + \frac{2000}{x}$

$\frac{dA}{dx} = 2x - 2000x^{-2} = 0$ gives $x^3 = 1000$, so $x = 10$

$\frac{d^2A}{dx^2} = 2 + 4000x^{-3} = 6 > 0$, so a minimum

A = 300

Answer: 300 m^2

7 Finding a percentage rate

Lena invests £5,500 in an account paying compound interest.
The interest rate is the same percentage each year.
After 3 years the account holds £7,320.50.
Work out the annual percentage interest rate.

Multiplier for 3 years = £7,320.50 ÷ £5,500 = 1.331
Yearly multiplier = the cube root of 1.331 = 1.1
10% increase each year
Answer: 10%

8 Transformations of graphs

The graph of y = f(x) has a minimum point at (0, −5).
Write down the coordinates of the turning point of the graph of
(a) $y = \frac{1}{2}f(x)$
(b) y = 2f(2x)

(a) Start with the turning point (0, −5)

Apply the transformation to the point: $(0, -\frac{5}{2})$

(b) f(2x) changes the x-coordinate to 0, and 2f(x) changes the y-coordinate to −10
Turning point: (0, −10)

Answer: (a) $(0, -\frac{5}{2})$ (b) (0, −10)

9 Wire cut in two

A piece of thin metal rod 136 cm long is cut into two pieces. One piece is bent to form a square of side x cm. The other piece is bent to form a rectangle whose length is twice its width.

(a) Show that the total area, A cm^2, enclosed by the two shapes is $A = x^2 + \frac{(136 - 4x)^2}{18}$.

(b) Find the value of x for which A is least, and work out this least area.

Rectangle perimeter $136 - 4x = 6w$, so $w = \frac{136 - 4x}{6}$ and its area is $2w^2 = \frac{(136 - 4x)^2}{18}$

$A = x^2 + \frac{(136 - 4x)^2}{18} = \frac{17}{9}x^2 - \frac{544}{9}x + \frac{9248}{9}$

$\frac{dA}{dx} = \frac{34}{9}x - \frac{544}{9} = 0$, so $x = 16$. $\frac{d^2A}{dx^2} = \frac{34}{9} > 0$, so a minimum

Square: 256, rectangle: $24 \times 12 = 288$, so A = 544

Answer: x = 16, least area 544 cm^2

10 Vectors: intersecting lines

OAB is a triangle. OA = **a** and OB = **b**.

M is on OA with OM : MA = 1 : 2. N is on OB with ON : NB = 1 : 4. AN and BM intersect at X.

(a) Find BM in terms of **a** and **b**.

(b) Find OX in terms of **a** and **b**.

The diagram is not drawn accurately.

(a) OM = $\frac{1}{3}$**a**, so BM = $-$**b** + $\frac{1}{3}$**a** = $\frac{1}{3}$**a** $-$ **b**

(b) X is on BM: BX = s BM, so OX = OB + s BM = $\frac{1}{3}$s**a** + (1 $-$ s)**b**

X is on AN: OX = OA + t AN = OA + t(ON $-$ OA) = (1 $-$ t)**a** + $\frac{1}{5}$t**b**

Equate: $\frac{1}{3}s = 1 - t$ and $1 - s = \frac{1}{5}t$

Solving: $s = \frac{6}{7}$ and $t = \frac{5}{7}$

OX = $\frac{2}{7}$**a** + $\frac{1}{7}$**b**

Answer: (a) $\frac{1}{3}$a $-$ b (b) OX = $\frac{2}{7}$a + $\frac{1}{7}$b