Day 113 **IGCSE 10-a-day: Model Solutions**

1 f′(x) notation

f(x) = $x^2 - 6x + 7$
(a) Find f′(x).
(b) Work out the value of f′(−3).

(a) $x^2 \to 2x$, $-6x \to -6$
f′(x) = $2x - 6$
(b) f′(−3) = −12
Answer: (a) f′(x) = 2x − 6 (b) −12

2 Laws of indices with algebra

Simplify fully $\frac{6x^{\frac{4}{3}} \times 2x^{-\frac{1}{2}}}{3x^{\frac{3}{2}}}$

Numbers: 6 × 2 ÷ 3 = 4

Add the powers for × and subtract for ÷: $\frac{4}{3} + -\frac{1}{2} - (\frac{3}{2}) = -\frac{2}{3}$

$= 4x^{-\frac{2}{3}}$

Answer: $4x^{-\frac{2}{3}}$

3 Gradient function from a given point

A curve has equation $y = -2x^3 + 6x^2 + 3$.
The point A (3, k) lies on the curve. Work out the value of k and the gradient of the curve at A.

Substitute x = 3 into the equation of the curve
y = 3 when x = 3, so k = 3
$\frac{dy}{dx} = -6x^2 + 12x$
At A: gradient = −18
Answer: k = 3, gradient = −18

4 Inverse of a rational function

f(x) = $\frac{-4x + 5}{3x - 4}$, $x \neq \frac{4}{3}$.

Find $f^{-1}(x)$.

$y = \frac{-4x + 5}{3x - 4}$, so $y(3x - 4) = -4x + 5$
$3xy - 4y = -4x + 5$
$3xy + 4x = 5 + 4y$
$x(3y + 4) = 5 + 4y$, so $x = \frac{5 + 4y}{3y + 4}$
$f^{-1}(x) = \frac{4x + 5}{3x + 4}$
Answer: $f^{-1}(x) = \frac{4x + 5}{3x + 4}$

5 3D Pythagoras and trigonometry: cuboids

The diagram shows a cuboid ABCDEFGH. (Calculator)
(a) Work out the length of the diagonal AG.
(b) Work out the angle between AG and the base ABCD, to 1 decimal place.
The diagram is not drawn accurately.

(a) $AC^2 = AB^2 + BC^2 = 20^2 + 5^2 = 425$
$AG^2 = AC^2 + CG^2 = 425 + 4^2 = 441$
AG = 21 cm
(b) The angle is GAC. $\tan(GAC) = \frac{CG}{AC} = \frac{4}{\sqrt{425}}$
angle GAC = 11.0° (1 d.p.)
Answer: (a) 21 cm (b) 11.0°

6 Stationary points of y = ax + b/x

$y = 2x + \frac{2}{x}$

Find the coordinates of the stationary points and determine whether each is a maximum or a minimum.

$\frac{dy}{dx} = 2 - \frac{2}{x^2}$

$\frac{dy}{dx} = 0$: $\frac{2}{x^2} = 2$, so $x^2 = 1$ and $x = 1$ or $x = -1$

$y = 4$ when $x = 1$ and $y = -4$ when $x = -1$

$\frac{d^2y}{dx^2} = \frac{4}{x^3}$

$x = 1$: $\frac{d^2y}{dx^2} = 4$ so minimum; $x = -1$: $\frac{d^2y}{dx^2} = -4$ so maximum

Answer: (1, 4) minimum; (−1, −4) maximum

7 Exponential graphs

The diagram shows the graph of $y = ka^x$, where k and a are constants and $a > 0$.
The curve passes through the points (1, 24) and (3, 384).
Find the value of k and the value of a.

Substitute the points: $k\,a^1 = 24$ and $k\,a^3 = 384$

Divide: $a^2 = \frac{384}{24} = 16$

$a = 4$ (positive, since $a > 0$)

From the first point: $k \times 4 = 24$, so $k = 6$

Answer: k = 6, a = 4

8 Exact lengths and areas

A rectangle has area $(1 + \sqrt{5})$ cm^2 and length $(2 + \sqrt{5})$ cm.
Find the width of the rectangle.
Give your answer in the form $a + b\sqrt{5}$

Width = area ÷ length = $\frac{1 + \sqrt{5}}{2 + \sqrt{5}}$

Multiply top and bottom by $(2 - \sqrt{5})$:

Numerator: $(1 + \sqrt{5})(2 - \sqrt{5}) = -3 + \sqrt{5}$

Denominator: $4 - 5 = -1$

Width = $3 - \sqrt{5}$

Answer: $3 - \sqrt{5}$ cm

9 Joint and inverse variation

P is directly proportional to the square of v and inversely proportional to the square of t.
When v = 3 and t = 1, P = 36.
Work out the value of P when v = 5 and t = 1.

$P = \frac{kv^2}{t^2}$

$36 = \frac{k \times 9}{1}$, so $k = 36 \times 1 \div 9 = 4$

$P = \frac{4 \times 25}{1} = 100$

Answer: P = 100

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10 Linear programming: vertices

x and y satisfy $x \geq 0$, $y \geq 0$,
$6x + 4y \leq 84$ and $2x + 4y \leq 44$.
Find the greatest value of $P = 5x + 7y$ and the values of x and y that give it.

Draw the lines. The feasible region has vertices (0, 0), (0, 11), (10, 6), (14, 0)
Values of P: (0, 0): 0, (0, 11): 77, (10, 6): 92, (14, 0): 70
Greatest value of P is 92 at (10, 6)
Answer: P = 92 when x = 10 and y = 6